Data Analysis and Probability

Chapter 12

Where You're Going

In this chapter, you will learn how to

- Use graphs to represent data.
- Find theoretical probability and experimental probability.
- Find permutations and combinations.
- Solve problems by doing simulations.

Real-World Snapshots Applying what you learn, on pages 684–685 you will solve problems about goal scoring in World Cup soccer.

LESSONS

Key Vocabulary

- box-and-whisker plot (p. 635)
- combination (p. 660)
- counting principle (p. 650)
- dependent events (p. 656)
- experimental probability (p. 665)
- frequency table (p. 630)
- independent events (p. 654)
- line plot (p. 631)
- permutation (p. 659)
- quartiles (p. 635)
- random sample (p. 669)
- range (p. 631)
- sample (p. 669)
- sample space (p. 650)
- simulation (p. 665)
- theoretical probability (p. 650)

12-1 Frequency Tables and Line Plots

What You'll Learn

OBJECTIVE 1 To display data in frequency tables

OBJECTIVE 2 To display data in line plots

. . . And Why

To solve real-world problems involving surveys

Check Skills You'll Need

Find the median and mode of each data set.

1. 6, 9, 9, 5, 9
2. 73, 78, 77, 73, 79
3. 300, 100, 200, 150, 300
4. 3, 5, 7, 9, 3, 4, 6, 3, 7

For help, go to Lesson 3-3.

New Vocabulary

- frequency table
- line plot
- range

TEXT Interactive lesson includes instant self-check, tutorials, and activities.

OBJECTIVE 1 Using Frequency Tables to Display Data

Investigation

Exploring Frequency Tables

Surveys Many people have favorite colors. Do people also have favorite numbers? Take a survey of your classmates.

1. Ask each person to choose an integer from 0 to 9. Use a table to record the responses.
2. Which number was chosen most frequently? How many times was each of the other numbers chosen?
3. Suppose you want to continue your survey by asking more people. Looking back, would you use the same type of table you used for Question 1? Can you make improvements? Explain.

You can display data in a **frequency table,** which lists each data item with the number of times it occurs.

1 EXAMPLE Building a Frequency Table

A number cube was rolled 20 times. The results are shown at the right. Display the data in a frequency table.

5	2	5	4	1	6	5	2	5	1
3	6	1	3	4	5	3	5	3	4

List the numbers on the cube in order. Use a tally mark for each result. Count the tally marks and record the frequency.

Number	Tally	Frequency
1	III	3
2	II	2
3	IIII	4
4	III	3
5	~~IIII~~ I	6
6	II	2

✔ Check Understanding Example 1

1. Display the data below in a frequency table.
   10 12 13 15 10 11 14 13 10 11 11 12 10 10 15

OBJECTIVE

2 Using Line Plots to Display Data

A **line plot** displays data with **X** marks above a number line.

The **range** of the data is the difference between the greatest and the least values in the data set.

> **Reading Math**
> This is the second use of the word *range*. The first appears in Lesson 8-1. You must read and use "range" *in context*, that is, according to the meanings of the words that appear with it.

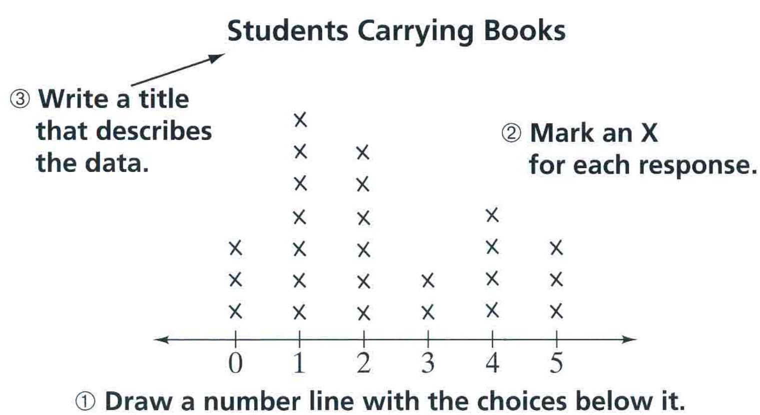
2 EXAMPLE Real-World Problem Solving

Surveys Twenty-five students in a school hallway were asked how many books they were carrying. The frequency table at the right shows their responses. Display the data in a line plot. Then find the range.

"How many books are you carrying?"

Number	Frequency
0	3
1	7
2	6
3	2
4	4
5	3

For a line plot, follow steps ①,②, and ③.



The greatest value in the data set is 5 and the least value is 0.
So the range is 5 − 0, or 5.

✔ Check Understanding Example 2

2. a. Display the data below in a line plot. Then find the range.
   miles from home to the mall: 2, 4, 3, 7, 3, 1, 4, 2, 2, 6, 3, 5, 1, 8, 3
   b. What is the range of the data below?
   prices of a gallon of regular gas at different gas stations:
   \$1.48, \$1.32, \$1.30, \$1.35, \$1.41, \$1.29, \$1.32, \$1.43, \$1.36

EXERCISES

For more exercises, see *Extra Practice.*

Practice and Problem Solving

A Practice by Example

Example 1 (page 630)

Display each set of data in a frequency table.

1. 1 4 0 3 0 1 3 2 2 4

2. 6 2 8 7 9 3 5 4 8 2 4 6 4 1

3. 10 30 20 30 50 10 40 30 50 40 30 50

4. 25 29 28 28 30 25 26 28 27 29 26 30

5. rolls of a number cube: 4 1 3 4 2 1 2 5 2 3 5 1 6 1 3 5 6

6. test scores: 100 90 70 60 95 65 85 70 70 75 80 85 75 70 100 90

Example 2 (page 631)

Draw a line plot for each frequency table.

7.

Number	1	2	3	4	5	6
Frequency	2	5	7	8	4	3

8.

Number	1	2	3	4	5	6
Frequency	1	3	5	8	8	5

In Exercises 9–14, display each set of data in a line plot. Find the range.

9. 0 2 1 1 4 0 4 3 2

10. 5 2 1 3 3 6 4 5 4 2

11. 5 0 2 1 4 3 4 0 2 5
4 3 2 0 4

12. 4 2 4 12 8 12 10 6 4
8 6 8 12

13. Literature the number of letters in each of the first twenty-five words of *Alice's Adventures in Wonderland* by Lewis Carroll:
4 3 10 5 3 9 2 3 4 5 2 7 2 3 6 2 3 4 3 2 6 7 2 2 4

B Apply Your Skills

14. the weekly earnings in dollars of the employees at Industrial Enterprises: 320, 320, 320, 400, 400, 400, 400, 400, 400, 480, 480, 480, 720, 720, 720, 1000

Display each set of data in a frequency table and in a line plot. Find the range.

15. ages of club members:
14 16 14 16 14 13 12 15 16 12 12 15 14 15 15

16. heights of plants (inches):
25 25 20 25 16 20 25 30 25 31 26 28 30

17. 7 11 10 10 8 11 9 7 9 8 11 11

18. 17 20 16 17 19 18 17 20 17 18 18 19 18 17

19. Baseball In the World Series, the first team to win four games is the champion. Sometimes the Series lasts for seven games, but sometimes the Series ends in fewer games. Below are data for 1970–2002. Make a frequency table and use it to find the mode.
Numbers of World Series Games, 1970–2002: 5, 7, 7, 7, 5, 7, 4, 6, 6, 7, 6, 6, 7, 5, 5, 7, 7, 7, 5, 4, 4, 7, 6, 6, 0, 6, 6, 7, 4, 4, 5, 7, 7

A frequency table or line plot may allow you to readily "see" the mode and find the median. Find the mode and the median for the data set in each exercise.

20. Exercise 11 **21.** Exercise 12 **22.** Exercise 13

23. Exercise 14 **24.** Exercise 15 **25.** Exercise 16

C Challenge

26. **Reasoning** A magazine line plot shows results of a survey. Explain how to use the line plot to find each of the following:
a. the number of people who answered the survey.
b. the mode, median, and mean

27. **Writing in Math** Describe a set of data that would be easier to display with a frequency table than with a line plot.

Test Prep

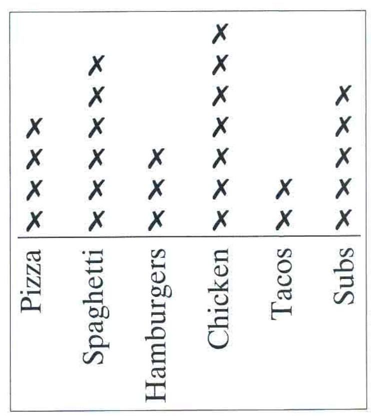
Multiple Choice

28. Below is a list of the ages of first-year teachers in one school system. What is the mode of the ages?
23, 42, 21, 25, 23, 24, 23, 24, 37, 23, 39, 51, 63, 24, 55
A. 23 **B.** 24 **C.** 37 **D.** 51

29. What is the range of the data below?
99.2, 101.5, 97.9, 102.1, 98.6, 100.4, 102.2, 99.9
F. 3.7 **G.** 3.9
H. 4.0 **I.** 4.3

Short Response

The line plot (right) represents the results of a class survey in which students were asked to name their favorite lunch.



30. **a.** How many students were surveyed?
b. How do you know?

31. **a.** What is the mode of the survey?
b. Explain your answer for part (a).

Take It to the NET
Online lesson quiz at
www.PHSchool.com
Web Code: ada-1201

Mixed Review

Lesson 11-7

32. **Measurement** The angle of elevation to a treetop from a point 10 ft out from the tree's base is 70°. Find the height of the tree.

Lesson 9-5

Given that $\triangle LMN \cong \triangle PQR$, complete each statement.

33. $\angle N \cong$ ■ **34.** $\overline{MN} \cong$ ■ **35.** $PQ =$ ■

Lesson 3-3

Find the mean, median, and mode for each data set.

36. 12 13 14 16 16 17 18 18 **37.** 8 15 22 9 11 16 20 10

Technology

Making Histograms

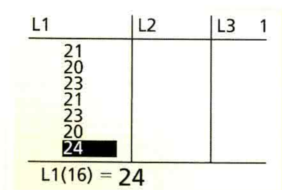
For Use With Lesson 12-1

A histogram shows the frequencies of data items as a graph. You can use a graphing calculator to make a histogram.

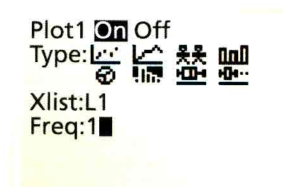
EXAMPLE

Make a histogram of the data below.
21, 23, 20, 22, 23, 21, 24, 26, 23, 21, 20, 23, 21, 23, 20, 24

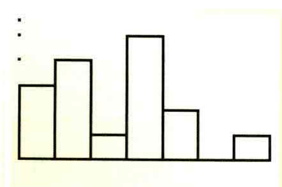
Step 1 Press **LIST** to find list L_1. Enter the data in L_1. (To first remove any data already in L_1, select L_1, then CLEAR ENTER.)



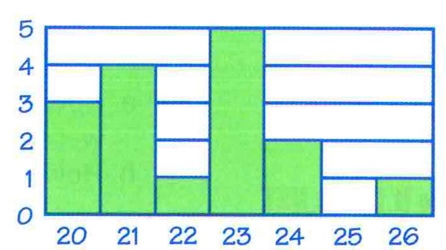
Step 2 In **PLOT**, enter 1 and select **On**. Select the Type of graph that looks like a histogram and ENTER.



Step 3 In the **ZOOM** menu, select item 7 "ZoomStat" and ENTER. In WINDOW, set Xscl = 1. Then GRAPH.



Step 4 To see the frequency of each number, press TRACE, and move the cursor across the histogram. Sketch the histogram.



EXERCISES

Use a graphing calculator to make a histogram of each set of data. Then sketch the histogram.

1. 11, 12, 12, 11, 10, 12, 13, 15, 9, 10, 12, 13
2. 9, 7, 6, 9, 8, 5, 9, 2, 2, 5, 8, 4, 6, 3, 8, 7, 8, 5
3. 23, 26, 25, 26, 23, 25, 25, 24, 21, 21, 22, 23
4. 95, 90, 92, 91, 95, 94, 93, 92, 94, 93, 95, 91
5. In Step 3 above, ENTER showed one histogram and then GRAPH showed another. They are histograms of the same data. Explain the difference in how they look.

Box-and-Whisker Plots

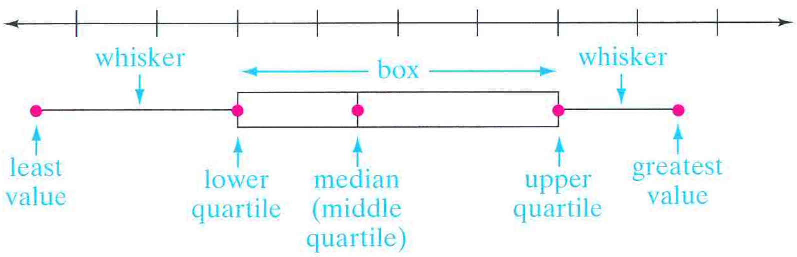
12-2

OBJECTIVE 1

Making Box-and-Whisker Plots

A **box-and-whisker plot** displays the distribution of data items along a number line. **Quartiles** divide the data into four equal parts. The median is the middle quartile.

Box-and-Whisker Plot



1 EXAMPLE **Real-World Problem Solving**

Statistics **The table, below right, shows United States crops harvested from 1988 to 2000. Make a box-and-whisker plot.**

Step 1 Arrange the data in order from least to greatest. Find the median.

298 308 314 317 318 318 321 322 323 326 326 327 333

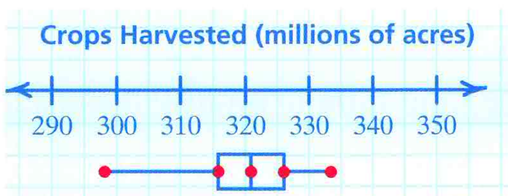
Step 2 Find the lower quartile and upper quartile, which are the medians of the lower and upper "halves."

298 308 314 317 318 318 321 322 323 326 326 327 333

lower quartile $= \frac{314 + 317}{2} = \frac{631}{2} = 315.5$

upper quartile $= \frac{326 + 326}{2} = \frac{652}{2} = 326$

Step 3 Draw a number line. Mark the least and greatest values, the median, and the quartiles. Draw a box from the first to the third quartiles. Mark the median with a vertical segment. Draw whiskers from the box to the least and greatest values.



What You'll Learn

OBJECTIVE 1 To make box-and-whisker plots

OBJECTIVE 2 To analyze data in box-and-whisker plots

. . . And Why

To solve real-world problems involving large data sets

Check Skills You'll Need

Find each median.

1. 12, 10, 11, 7, 9, 8, 10, 5
2. 4.5, 3.2, 6.3, 5.2, 5, 4.8, 6, 3.9
3. 55, 53, 67, 52, 50, 49, 51, 52, 52, 52
4. 101, 100, 100, 105, 102, 101

For help, go to Lesson 3-3.

New Vocabulary

- box-and-whisker plot
- quartiles

Crops Harvested

Year	Acres (millions)	Year	Acres (millions)
1988	298	1995	314
1989	318	1996	326
1990	322	1997	333
1991	318	1998	326
1992	317	1999	327
1993	308	2000	323
1994	321		

SOURCE: *Statistical Abstract of the United States.* Go to **www.PHSchool.com** for a data update. Web Code: adg-2041

TEXT Interactive lesson includes instant self-check, tutorials, and activities.

Check Understanding Example 1

1. Draw a box-and-whisker plot for the distances of migration of birds (thousands of miles): 5, 2.5, 6, 8, 9, 2, 1, 4, 6.2, 18, 7.

You can compare two sets of data by making two box-and-whisker plots below one number line.

Real-World Connection

DNA evidence suggests that whales and hippopotamuses are closely related genetically.

2 EXAMPLE Real-World Problem Solving

Biology **Use box-and-whisker plots to compare orca whale masses and hippopotamus masses.**

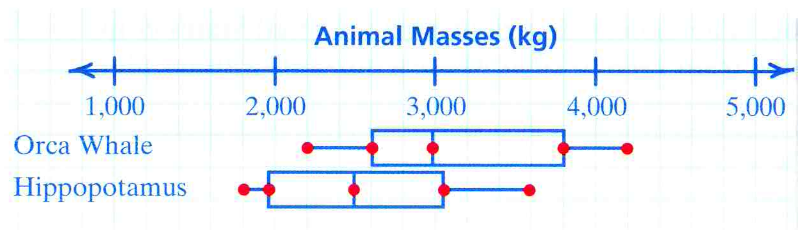
Orca whale masses (kg)

3,900 2,750 2,600 3,100 4,200 2,600 3,700 3,000 2,200

Hippopotamus masses (kg)

1,800 2,000 3,000 2,500 3,600 2,700 1,900 3,100 2,300

Draw a number line for both sets of data. Use the range of data points to choose a scale.



Draw the second box-and-whisker plot below the first one.

Check Understanding Example 2

2. Compare annual video sales and CD sales by making two box-and-whisker plots below one number line.

   videos (millions of units): 28, 24, 15, 21, 22, 16, 22, 30, 24, 17

   CDs (millions of units): 16, 17, 22, 16, 18, 24, 15, 16, 25, 18

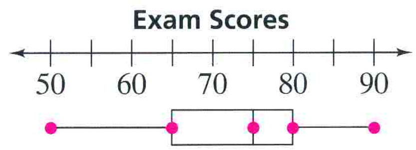
OBJECTIVE

2 Analyzing Box-and-Whisker Plots

Although you cannot see every data point in a box-and-whisker plot, you can use the quartiles and the greatest and least values to analyze and describe a data set.

3 EXAMPLE Describing Data

Describe the data in the box-and-whisker plot.

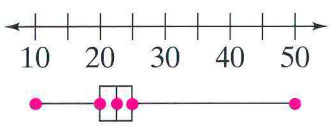


The highest score is 90 and the lowest is 50. At least half of the scores are within 10 points of the median, 75.

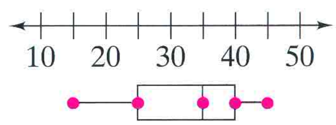
✓ Check Understanding Example 3

3. Describe the data in each box-and-whisker plot.

a.



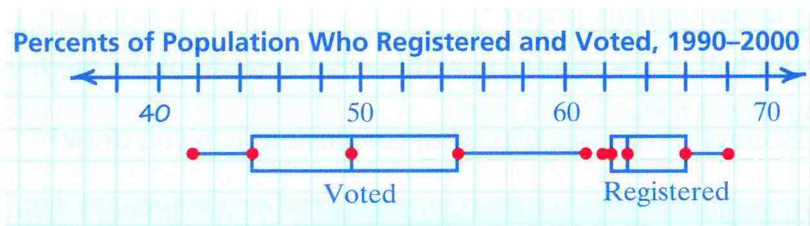
b.



You can compare box-and-whisker plots to analyze two sets of data.

4 EXAMPLE Real-World Problem Solving

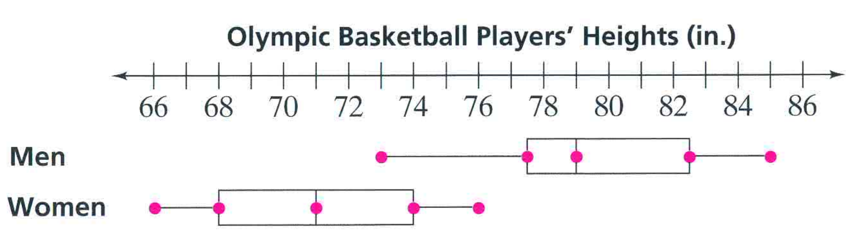
Social Studies **The plots below compare the percents of the voting-age population who said they registered to vote in U.S. elections to the percents who said they voted. What conclusions can you draw?**



The percent registered was fairly constant, since the box-and-whisker plot is narrow. The percent who voted varied more. You can conclude that in an election, on average, the percent of people who voted was about 15 less than the percent of people who were registered.

✓ Check Understanding Example 4

4. Use box-and-whisker plots below. What conclusions can you draw about heights of Olympic basketball players?



12-3 Using Graphs to Persuade

What You'll Learn

OBJECTIVE 1 To recognize the use of breaks in the scales of graphs

OBJECTIVE 2 To recognize the use of different scales

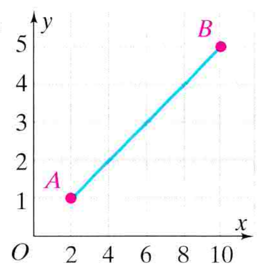
. . . And Why

To solve real-world problems involving population and cost of living

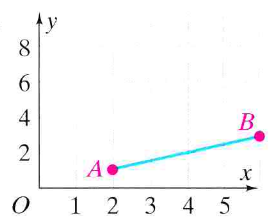
Check Skills You'll Need

Find the slope of $\overline{AB}$ in each graph.

1.



2.



For help, go to Lesson 8-3.

OBJECTIVE

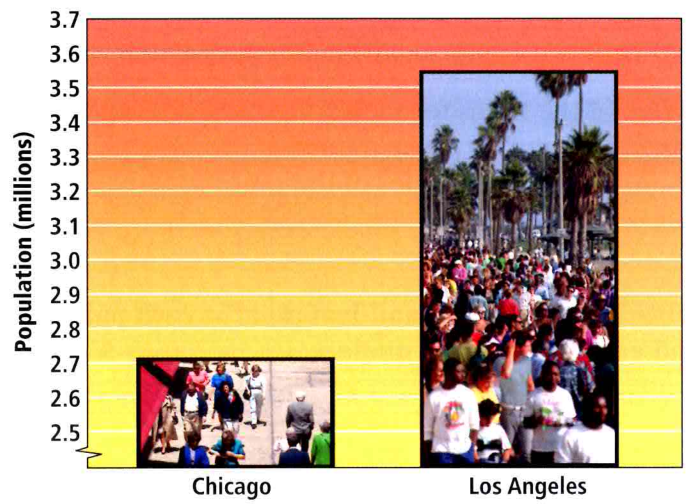
1 Using Breaks in Scales

You can draw graphs of a data set in different ways in order to give different impressions.

You can use a break in the scale on one or both axes of a line graph or a bar graph. This lets you show more detail and emphasize differences. However, it can also give a distorted picture of the data.

1 EXAMPLE Real-World Problem Solving

Population **Which title would be more appropriate for the graph below: "Los Angeles Overwhelms Chicago" or "Populations of Chicago and Los Angeles"? Explain.**



Because of the break in the vertical axis, the bar for Los Angeles appears to be more than three times as tall as the bar for Chicago. Actually, the population of Los Angeles is a little less than 3.6 million, and the population of Chicago is about 2.7 million. So the population of Los Angeles is about 1.3 times that of Chicago.

The title "Los Angeles Overwhelms Chicago" could be misleading. "Populations of Chicago and Los Angeles" better describes the information in the graph.

Check Understanding Example 1

1. Use the data in the graph in Example 1. Redraw the graph without a break.

TEXT Interactive lesson includes instant self-check, tutorials, and activities.

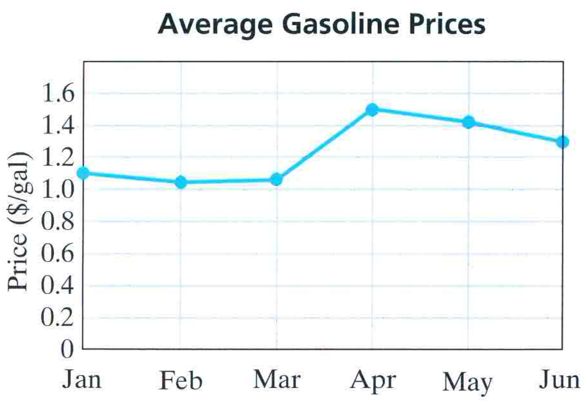
OBJECTIVE

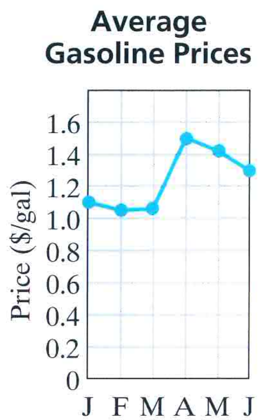
2 Using Different Scales

You can use different scales or spacing along axes. This lets you emphasize (or de-emphasize) how changes in data are related.

2 EXAMPLE Real-World Problem Solving

Cost of Living **Study the graphs below. Which graph gives the impression of a sharper increase in price? Explain.**





In the graph at the right, the months are much closer together, so the line appears to climb more rapidly. This graph suggests that prices are going up faster than suggested by the graph at the left.

Check Understanding Example 2

2. Use the data in the table at the right.
   a. Make a graph that suggests a rapid decrease in the total weight of fish caught.
   b. Make a graph that suggests a slow decrease in the total weight of fish caught.
   c. **Reasoning** A group is planning a campaign to protect the supply of fish. They are proposing a regulation that would limit the number of pounds of fish caught annually. Would they more likely use the graph from part (a) or part (b) in their proposal? Explain.

Fish Caught for Food in the U.S.

Year	Fish Caught (billions of pounds)
1993	8.2
1994	7.9
1995	7.7
1996	7.5

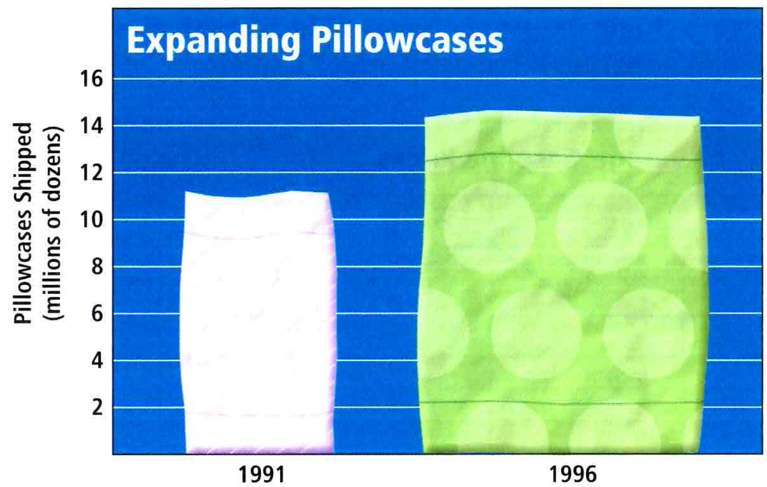
SOURCE: *Statistical Abstract of the United States.* Go to **www.PHSchool.com** for a data update. Web Code: adg-2041

There is a fine line between using graphs to persuade and using graphs to mislead.

Bar graphs can be misleading if their "bars" change in more than one dimension. This can happen when graphs use realistic images for the bars. These images make the graphs more interesting but also can give false impressions.

3 EXAMPLE Real-World Problem Solving

Reasoning **What makes the graph misleading? Explain.**



By reading the vertical axis, you can see that the number of pillowcases shipped increased by about one fourth.

However, the bar on the right has not only increased in height, but has also nearly doubled in width. The area of the second bar is more than two times the area of the first bar.

Because of this, you might get the impression that the increase was much greater than it really was.

Check Understanding Example 3

3. Use the data in the table below.

Prices of Field-Grown Tomatoes in the United States

Year	Price of Tomatoes (cents per pound)
1990	86
1997	162

SOURCE: *Statistical Abstract of the United States*. Go to **www.PHSchool.com** for a data update. Web Code: adg-2041

a. Draw a graph that suggests that the price of tomatoes nearly doubled.

b. Draw a graph that suggests that the price of tomatoes more than doubled.

EXERCISES

For more exercises, see *Extra Practice.*

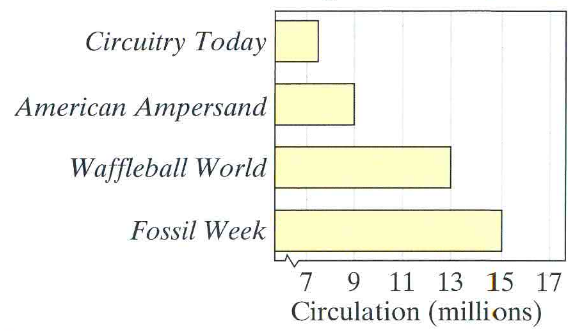
Practice and Problem Solving

A Practice by Example

Example 1 (page 642)

For Exercises 1–4, use the graph at the right.

Magazine Circulation

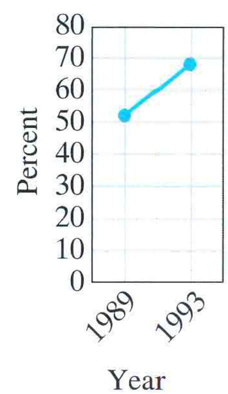


1. Which magazine *appears* to have about twice the circulation of *Circuitry Today?*
2. Which magazine *actually* has twice the circulation of *Circuitry Today?*
3. Explain why the graph might mislead you.
4. Redraw the graph without a break in the horizontal axis.

Example 2 (page 643)

Statistics **For Exercises 5–7, use the graph at the right.**

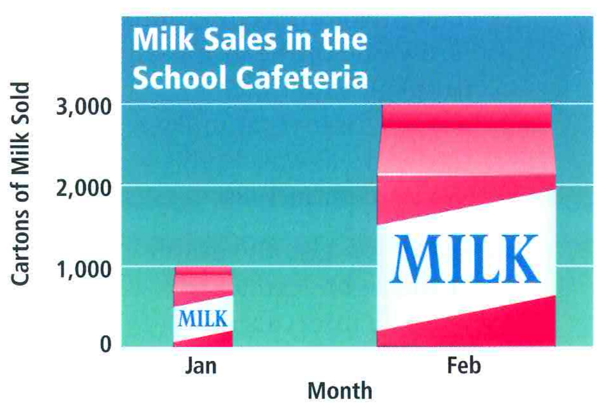
Percent of Students Using Computers at School



5. **School Computers** Does the graph suggest a rapid increase or a slow increase in the percent of students using a computer at school?
6. Redraw the graph. Change the horizontal scale to suggest a slower increase from 1989 to 1993.
7. Redraw the graph. Suggest a slower increase from 1989 to 1993 by changing the vertical scale.

Example 3 (page 644)

Food Service **For Exercises 8–10, use the graph below.**



8. What impression does the graph give you about milk sales in the school cafeteria? Is the graph misleading? Explain.
9. Redraw the graph to represent the data accurately.
10. Redraw the graph to suggest that milk sales changed very little.

B Apply Your Skills

Reading Math

For help with reading the table and drawing the graph in Exercise 11, see page 648.

11. a. **Statistics** Use the data at the right. Draw a graph with an axis break to suggest that enrollment in 2000 was many times the enrollment in 1990.

b. Draw a second graph of the data, without using a break. Choose a scale that suggests that enrollment did not increase much from 1990 to 2000.

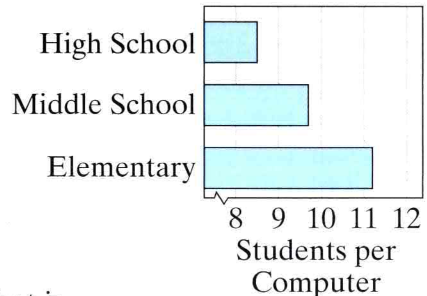
U.S. College Enrollment

Year	Enrollment
1990	13.8 million
1995	14.2 million
2000	14.9 million

SOURCE: U.S. Education Department

For Exercises 12–14, use the graph at the right.

Average Number of Students per Computer



12. **Writing in Math** The graph suggests that the number of students per computer in elementary schools is three times the number of students per computer in high schools. Is this true? Explain.

13. What does the graph suggest is the ratio of middle school students per computer to high school students per computer? What is the actual ratio?

14. Redraw the graph without a break. Describe the effect this has on what the graph suggests.

Average Annual Tuition and Fees for Four-Year Public Colleges

Year	Tuition and Fees
1980–1981	\$1,647
1990–1991	\$2,529
2000–2001	\$3,535

SOURCE: The College Board

15. Use the data at the left. Draw a line graph that gives the impression of a gradual increase in college tuition and fees from 1980 to 2001.

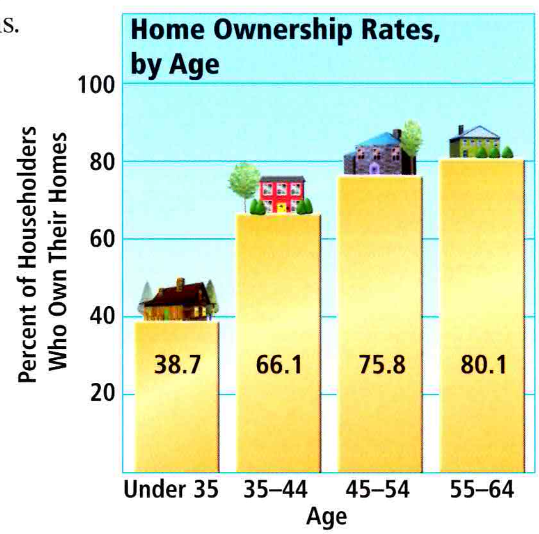
16. **Open-Ended** Find a graph in a newspaper or magazine that could be misleading. Explain how it could be misleading.

C Challenge

Percent of Milk Sold That Was Low-Fat

Year	Percent
1980	38%
1990	59%

17. Use the data at the left to make two different graphs. Draw one of the graphs to suggest that the percent of low-fat milk sold in 1990 was double the percent in 1980.



18. **Statistics** Use the graph at the right. Explain why the intervals on the horizontal axis could make the graph misleading.

19. Tell how to scale the x- and y-axes so that $\overline{AB}$ joining points $A(4, 2)$ and $B(8, 6)$ appears to have the slope given.

a. $\frac{1}{2}$ **b.** 1 **c.** 2

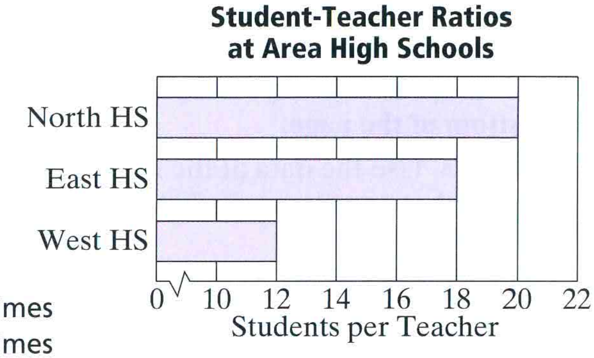
Test Prep

Multiple Choice

For Exercises 20 and 21, use the graph below.

20. The graph makes it appear that there are how many times as many students per teacher at North HS as there are at West HS?

A. 2 times **B.** 3 times
C. 4 times **D.** 5 times



21. Why might this graph give a distorted picture of the data?
- **F.** The longest horizontal bar is on top.
- **G.** The vertical lines are evenly spaced.
- **H.** There is a break in the horizontal axis.
- **I.** The horizontal bars are different lengths.

Short Response

22. Use the table at the right to make two different line graphs.
- **a.** Draw one graph to suggest that sales more than doubled from 1996 to 1999.
- **b.** Draw the other graph to suggest that sales increased only slightly during the same period.

Annual Sales

Year	Sales
1996	\$87 million
1997	\$87 million
1998	\$88 million
1999	\$90 million

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Web Code: ada-1203

Mixed Review

Lesson 12-2

Make a box-and-whisker plot for each set of data.

23. 27, 25, 23, 29, 25, 28, 26, 27, 23, 21, 20, 24, 25, 28, 30, 19, 25

24. 2, 6, 3, 9, 15, 4, 9, 20, 6, 7, 2, 3, 8, 4, 1, 5, 6, 8, 5, 4, 9, 3, 2, 8, 7

25. 100, 95, 102, 101, 96, 100, 104, 115, 102, 108, 92, 97, 103, 106

Lessons 10-5

26. Geometry The Museum of Health and Medical Science in Houston, Texas, has one of the largest kaleidoscopes in the world. It is a cylinder 10 feet long and 22 inches in diameter. What is the surface area of the kaleidoscope?

Lesson 6-4

Find each probability for choosing a letter at random from the word STATISTICS.

27. P(vowel) **28.** P(S) **29.** P(not T) **30.** P(A or C)

Reading Math

Reading for Problem Solving

For Use With Page 646, Exercise 11

Read the problems below. Then follow along with what Kayla thinks as she solves them. Check your understanding by solving the exercise at the bottom of the page.

a. **Statistics** Use the data at the right. Draw a graph with a break to suggest that enrollment in 2000 was many times the enrollment in 1990.

b. Draw a second graph of the data without using a break. Choose a scale that suggests that enrollment did not increase much from 1990 to 2000.

U.S. College Enrollment

Year	Enrollment
1990	13.8 million
1995	14.2 million
2000	14.9 million

SOURCE: U.S. Education Department

What Kayla Thinks

Part (a) asks me to draw a graph with a break that makes enrollment in 2000 look *many times* the enrollment in 1990.

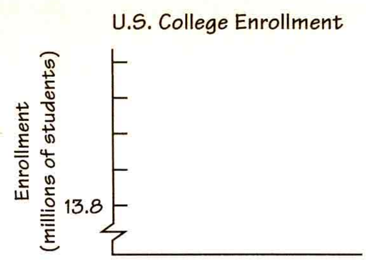
I'll use a bar graph. The table shows the 1990 enrollment was 13.8 million. I'll break the vertical scale right before 13.8 to make the 1990 bar look small.

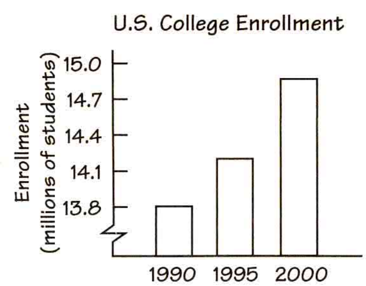
The table shows a 2000 enrollment of 14.9 million. I'll use vertical intervals of 0.3. That makes the top mark 15.0.

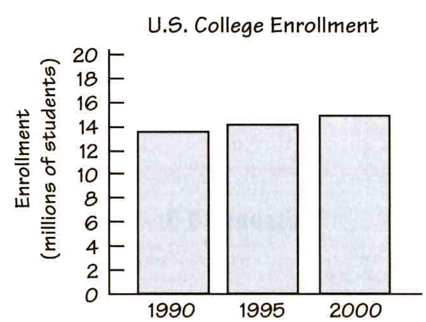
The bar for 14.9 is as tall as possible. It looks nearly 4 times as tall as 1990!

Part (b) asks for no break. I'll scale by 2 on the vertical axis. This will make 1990 and 2000 almost the same height.

What Kayla Draws







EXERCISE

1. Two months ago, the price for Technostar stock was $32. Last month it was $32.40. Now it is $32.80. Draw graphs suggesting each of the following about the stock price.

   a. a large increase   **b.** a small increase

Counting Outcomes and Theoretical Probability

12-4

OBJECTIVE 1

Counting Possible Choices

Investigation

Exploring Possible Outcomes

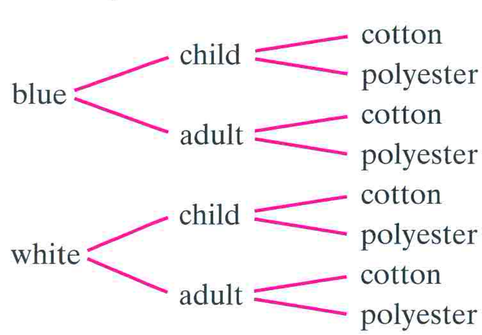
Congratulations! Your application to run the pizza stand at school home games has been accepted. Now you have to decide which pizzas to sell. You plan to offer two or three choices in each of three categories—size, crust, and topping. The more types of pizza the better, but you're limited by kitchen space to a total of 18 types.

1. Decide which types of pizza you will offer. Make a menu that shows your customers their options.
2. **Reasoning** Suppose you decide to offer three choices of size and three choices of crust. How many choices of toppings can you offer?

You can use a tree diagram to display and count possible choices.

1 EXAMPLE Drawing a Tree Diagram

A school team sells caps in two colors (blue or white), two sizes (child or adult), and two fabrics (cotton or polyester). Draw a tree diagram to find the number of cap choices.



Each branch of the "tree" represents one choice—for example, blue-child-cotton.

There are 8 possible cap choices.

Check Understanding Example 1

1. Suppose the caps in Example 1 also come in black. Draw a tree diagram. How many cap choices are there?

What You'll Learn

OBJECTIVE 1 To use a tree diagram and the Counting Principle to count possible choices

OBJECTIVE 2 To find theoretical probability by counting outcomes

. . . And Why

To solve real-world problems involving probabilities and outcomes

Check Skills You'll Need

A bag has 5 blue (B) chips, 4 red (R) chips, and 3 tan (T) chips. Find each probability for choosing a chip at random from the bag.

1. $P(\text{R})$
2. $P(\text{not R})$
3. $P(\text{B})$
4. $P(\text{R or B})$
5. $P(\text{T})$
6. $P(\text{B or T})$

For help, go to Lesson 6-4

New Vocabulary

- counting principle
- theoretical probability
- sample space

iTEXT Interactive lesson includes instant self-check, tutorials, and activities.

Another way to count choices is to use the **Counting Principle.**

> **Test-Taking Tip**
> The Counting Principle is sometimes called the "Multiplication Counting Principle."

Key Concepts — Counting Principle

If there are m ways of making one choice, and n ways of making a second choice, then there are $m \cdot n$ ways of making the first choice followed by the second.

The Counting Principle is particularly useful when a tree diagram would be too large to draw.

Real-World Connection

A monogram is made up of two or more letters, such as your initials.

2 EXAMPLE — Using the Counting Principle

How many two-letter monograms are possible?

first letter		second letter		monograms
possible choices		possible choices		possible choices
26	·	26	=	676

There are 676 possible two-letter monograms.

Check Understanding Example 2

2. a. How many three-letter monograms are possible?
   b. How many five-letter license plates can be made if the letters O and I cannot be used?

OBJECTIVE 2 Finding Probability by Counting Outcomes

You can count outcomes to help you find the **theoretical probability** of an event in which outcomes are equally likely.

Key Concepts — Theoretical Probability

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

A **sample space** is a list of all possible outcomes. You can use a tree diagram to find a sample space. Then you can calculate probability.

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3 EXAMPLE Using a Tree Diagram

Use a tree diagram to find the sample space for tossing two coins. Then find the probability of tossing two tails.

heads — heads
heads — tails
tails — heads
tails — tails

The tree diagram shows there are four possible outcomes, one of which is tossing two tails.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$ **Use the probability formula.**

$$P(\text{two tails}) = \frac{\text{number of two-tail outcomes}}{\text{number of possible outcomes}}$$

$$= \frac{1}{4}$$

The probability of tossing two tails is $\frac{1}{4}$.

Check Understanding Example 3

3. You toss two coins. Find P(one head and one tail).

You can also use the Counting Principle to find probability.

4 EXAMPLE Real-World Problem Solving

Many people play lottery games without knowing the probability of winning. In some state lotteries, the winning number is made up of four digits chosen at random. Suppose a player buys two tickets with different numbers. What is the probability that the player has a winning ticket?

First find the number of possible outcomes. For each digit, there are 10 possible outcomes, 0 through 9.

1st digit outcomes		2nd digit outcomes		3rd digit outcomes		4th digit outcomes		total outcomes
10	·	10	·	10	·	10	=	10,000

Then find the probability when there are two favorable outcomes.

$$P(\text{winning ticket}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \frac{2}{10{,}000}$$

The probability is $\frac{2}{10{,}000}$, or $\frac{1}{5{,}000}$.

Check Understanding Example 4

4. A lottery uses five digits chosen at random. Find the probability of buying a winning ticket.

EXERCISES

For more exercises, see *Extra Practice.*

Practice and Problem Solving

A Practice by Example

Example 1 (page 649)

You can choose a burrito having one filling wrapped in one tortilla. Draw a tree diagram to count the number of burrito choices.

1. tortillas: flour or corn; fillings: beef, chicken, bean, cheese, or vegetable
2. tortillas: whole wheat flour, blue corn, or white corn; fillings: chicken, tofu, grilled fish, or vegetable

Example 2 (page 650)

There are four roads from Marsh to Taft and seven roads from Taft to Polk. Use the Counting Principle to find the number of routes below.

3. from Marsh to Polk through Taft
4. from Marsh to Polk after a new road opens from Marsh to Taft

Example 3 (page 651)

Use a tree diagram to find the sample space for tossing three coins. Then find each probability.

5. P(three heads)
6. P(two tails)
7. P(at least one head)

Example 4 (page 651)

Use the Counting Principle to help you find each probability.

8. Rolling a 3 on each of two number cubes
9. **Lottery** Choosing the three winning lottery numbers when the numbers are chosen at random from 1 to 50. Numbers can repeat.

B Apply Your Skills

10. **Snacks** You can choose chocolate, strawberry, or vanilla frozen yogurt, and red, blue, or green sprinkles. A sundae has one yogurt flavor and two different colors of sprinkles. How many different kinds of sundaes can you order? List them.

Sweaters

Colors	Styles
Blue	Cardigan
Pink	Crewneck
Red	V-neck
Brown	
Black	

You have one sweater of each possible color and style in the table (left).

11. How many sweaters do you have?
12. What is the probability of choosing a brown sweater at random?
13. What is the probability of choosing a cardigan at random?

Find the probability of each event.

14. You toss tails and roll an even number (when you toss a coin and roll a number cube).
15. You roll two odd numbers and pick a vowel (when you roll two number cubes and pick a letter of the alphabet at random).

C Challenge

16. **Reasoning** You have a bag containing an equal number of nickels, dimes, and quarters. You reach into the bag and choose a coin. Are all outcomes equally likely? Explain.
17. **Writing in Math** Write a problem (unlike any in this lesson) that you can solve using the Counting Principle. Then solve.

Test Prep

Multiple Choice

18. You are writing a three-digit number. The first digit must be 2 or 8. The second digit must be 1, 3, or 9. The third digit must be 4, 5, 6, 7, or 8. Which expression can you use to find how many different three-digit numbers you can write?

A. $2 \cdot 3 \cdot 5$ **B.** $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
C. $(1 \cdot 2) + (2 \cdot 3) + (3 \cdot 5)$ **D.** $2 + 3 + 5$

19. An ice-cream vendor sells small or large cones and has chocolate, vanilla, rocky road, pecan, and strawberry flavors. To help a customer select a flavor and size, the vendor has a spinner. What is the probability that the spinner will choose a large strawberry for a customer?

F. $\frac{1}{14}$ **G.** $\frac{1}{10}$ **H.** $\frac{1}{7}$ **I.** $\frac{1}{3}$

Extended Response

20. You roll two number cubes. List all possible outcomes. Find the probability of each event. Show your work.

a. rolling a 1 and a 2
b. rolling the same numbers
c. rolling different numbers

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Mixed Review

Lesson 12-1 **Display each data set in a line plot. Find the range.**

21. 3 4 5 4 7 7 3 6 5 **22.** 19 18 19 17 17 16 19 18 17 19

Lesson 11-3 **Find the midpoint of a segment with the given endpoints.**

23. $X(3, -2)$ and $Y(-3, 6)$ **24.** $A(-1, 0)$ and $B(2, 1)$

Checkpoint Quiz 1 — Lessons 12-1 through 12-4

iTEXT Instant self-check quiz online and on CD-ROM

1. Display the data below in a frequency table.
47 51 50 52 50 47 48 50 49 51 48 52

2. Make a box-and-whisker plot for the data below.
31, 33, 74, 90, 44, 49, 40, 64, 42, 31, 36, 73, 86, 34, 46, 65

3. **Open-Ended** Use the data in the table.

a. Draw a graph that could be misleading. Explain.
b. Draw a second graph that is not misleading.

Year	Hourly Minimum Wage
1996	\$4.75
1997	\$5.15

SOURCE: *Wall Street Journal Almanac*

4. Olivia and Oliver each choose a number from 1 to 10 at random. What is the probability that both numbers are odd numbers?

12-5 Independent and Dependent Events

What You'll Learn

OBJECTIVE 1 To calculate probabilities of independent events

OBJECTIVE 2 To calculate probabilities of dependent events

. . . And Why

To solve real-world problems involving games and science

Check Skills You'll Need

Multiply.

1. $\frac{3}{5} \cdot \frac{1}{5}$
2. $\frac{1}{4} \cdot \frac{2}{4}$
3. $\frac{4}{10} \cdot \frac{2}{10}$
4. $\frac{5}{9} \cdot \frac{4}{8}$
5. $\frac{4}{7} \cdot \frac{3}{6}$
6. $\frac{9}{10} \cdot \frac{8}{9}$

For help, go to Lesson 5-4.

New Vocabulary

- independent events
- dependent events

Interactive lesson includes instant self-check, tutorials, and activities.

OBJECTIVE 1 Independent Events

Investigation: Exploring Probability in Games

You have four cards with an M written on them, two with an A, six with a T, and eight with an H.

1. You draw an M card at random and replace it. What is the probability that the next card you draw at random will also be an M card?
2. You draw an M card at random and do not replace the card. What is the probability that the next card you draw at random will also be an M card?
3. Make a table to find the probability of matching cards.

Probability *With* Replacement

First Card	Second Card Matches
$P(M) = ■$	$P(M) = ■$
$P(A) = ■$	$P(A) = ■$
$P(T) = ■$	$P(T) = ■$
$P(H) = ■$	$P(H) = ■$

Probability *Without* Replacement

First Card	Second Card Matches
$P(M) = ■$	$P(M) = ■$
$P(A) = ■$	$P(A) = ■$
$P(T) = ■$	$P(T) = ■$
$P(H) = ■$	$P(H) = ■$

4. **Reasoning** For any letter, why is the probability for selecting the second card with replacement of the first card different from the probability of selecting the second card without replacement of the first card?

Suppose the numbers from 1 to 10 are written on 10 cards, one number to a card. You are interested in drawing one card at random and getting an even number, and then drawing a second card and getting an even number again.

If you *replace* your first card, the probability of getting an even number on the second card is unaffected.

Independent events are events for which the occurrence of one event *does not affect* the probability of the occurrence of the other.

Key Concepts — Probability of Independent Events

For two independent events A and B, the probability of both events occurring is the product of the probabilities of each event occurring.

$$P(A, \text{then } B) = P(A) \cdot P(B)$$

1 EXAMPLE — Finding Probability for Independent Events

You roll a number cube once. Then you roll it again. What is the probability that you get 2 on the first roll and a number greater than 4 on the second roll?

$P(2) = \frac{1}{6}$ — **There is one 2 among 6 numbers on a number cube.**

$P(\text{greater than } 4) = \frac{2}{6}$ — **There are two numbers greater than 4 on a number cube.**

$$\begin{aligned} P(2, \text{then greater than } 4) &= P(2) \cdot P(\text{greater than } 4) \\ &= \frac{1}{6} \cdot \frac{2}{6} \\ &= \frac{2}{36}, \text{ or } \frac{1}{18} \end{aligned}$$

The probability is $\frac{1}{18}$.

Check Understanding — Example 1

1. You toss a coin twice. Find the probability of getting two heads.

You can use fractions, decimals, or percents to represent probabilities and to find the probability of two events occurring.

2 EXAMPLE — Real-World Problem Solving

Botany **Under the best conditions, a wild bluebonnet seed has a 20% probability of growing. If you select two seeds at random, what is the probability that both will grow, under the best conditions?**

$P(\text{a seed grows}) = 20\%, \text{ or } 0.20$ — **Write the percent as a decimal.**

$$\begin{aligned} P(\text{two seeds grow}) &= P(\text{a seed grows}) \cdot P(\text{a seed grows}) \\ &= 0.20 \cdot 0.20 && \textbf{Substitute.} \\ &= 0.04 && \textbf{Multiply.} \\ &= 4\% && \textbf{Write 0.04 as a percent.} \end{aligned}$$

The probability that two seeds grow is 4%.

Real-World Connection

Bluebonnets grow wild in the southwestern United States.

Check Understanding Example 2

2. **Botany** Chemically treated bluebonnet seeds have a 30% probability of growing. You select two such seeds at random. What is the probability that both will grow?

OBJECTIVE

2 Dependent Events

Suppose you want to draw two even-numbered cards from cards showing numbers from 1 to 10. You draw one card. Then, *without replacing* the first card, you draw a second card. The probability of drawing an even number on the second card is affected.

Dependent events are events for which the occurrence of one event *affects* the probability of the occurrence of the other.

Key Concepts Probability of Dependent Events

For two dependent events A and B, the probability of both events occurring is the product of the probability of the first event and the probability that, after the first event, the second event occurs.

$$P(A, \text{then } B) = P(A) \cdot P(B \text{ after } A)$$

3 EXAMPLE Finding Probability for Dependent Events

Three girls and two boys volunteer to represent their class at a school assembly. The teacher selects one name and then another from a bag containing the five students' names. What is the probability that both representatives will be girls?

$P(\text{girl}) = \frac{3}{5}$ — **Three of five students are girls.**

$P(\text{girl after girl}) = \frac{2}{4}$ — **If a girl's name is drawn, two of the four remaining students are girls.**

$$\begin{aligned} P(\text{girl, then girl}) &= P(\text{girl}) \cdot P(\text{girl after girl}) \\ &= \tfrac{3}{5} \cdot \tfrac{2}{4} && \textbf{Substitute.} \\ &= \tfrac{6}{20}, \text{ or } \tfrac{3}{10} && \textbf{Simplify.} \end{aligned}$$

The probability that both representatives will be girls is $\frac{3}{10}$.

Check Understanding Example 3

3. a. For Example 3, find $P(\text{boy, then girl})$.
   b. Find $P(\text{girl, then boy})$.

EXERCISES

For more exercises, see *Extra Practice.*

Practice and Problem Solving

A Practice by Example

Example 1 (page 655)

You roll a number cube twice. What is the probability that you roll each pair of numbers?

1. 6, then 5
2. 6, then a number less than 4
3. 6, then 2 or 5
4. an even number, then 2 or 5
5. 1, then 1
6. an even number, then an odd number

Example 2 (page 655)

7. **Weather Forecasting** Weather forecasters are accurate 91% of the time when predicting precipitation for the day. What is the probability that a forecaster will make correct precipitation predictions two days in a row?

Example 3 (page 656)

You select a card at random from those below. Without replacing the card, you select a second card. Find the probability of selecting each set of letters.

P R E A L G E B R A

8. P, then G
9. E, then A
10. E, then a second vowel
11. G, then R or A
12. P or E, then A
13. a consonant, then a vowel

B Apply Your Skills

Are the events independent or dependent? Explain.

14. You select a card. Without putting the card back, you select a second card.
15. You roll a number cube. You roll it again.

You pick a marble from a bag containing 1 green marble, 4 red marbles, 2 yellow marbles, and 3 black marbles. You replace the first marble and then select a second one. Find each probability.

16. P(red, then yellow)
17. P(black, then black)
18. P(red, then black)
19. P(yellow, then black)

Gino has 5 blue socks and 4 black socks. He selects one sock at random. Without replacing the sock, he selects a second sock at random. Find each probability.

20. P(blue, then black)
21. P(black, then blue)
22. P(black, then black)
23. P(blue, then blue)

24. **Writing in Math** Explain the difference between independent and dependent events.

25. A refrigerator contains 12 orange drinks, 4 grape drinks, and 25 apple drinks. Ann is first in the line for drinks. Mark is second. What is the probability that Ann gets an apple drink and Mark gets a grape drink, if they are given drinks at random?

C Challenge

26. On a multiple-choice test you randomly guess the answers to two questions. Each question has five choices.
   a. What is the probability that you get both answers correct?
   b. What is the probability that you get both answers incorrect?

27. Mrs. Kendall's wallet contains 3 one-dollar bills, 2 five-dollar bills, and 3 ten-dollar bills. She randomly selects one bill and then another. Find the probability that she selects the given bills.
   a. a one-dollar bill and then a ten-dollar bill
   b. a ten-dollar bill and then a five-dollar bill

Test Prep

Multiple Choice

28. There are 6 girls and 5 boys in a debate class. The teacher chooses two students at random to lead a class discussion. What is the probability of selecting a girl and then a boy?

   A. $\frac{30}{121}$ **B.** $\frac{3}{11}$ **C.** $\frac{36}{121}$ **D.** $\frac{6}{11}$

29. Mike asks Carolyn to pick a number at random from 1 to 100, and then asks Jamie to do the same. What is the probability that both girls will select 49?

   F. $\frac{1}{50}$ **G.** $\frac{1}{100}$ **H.** $\frac{1}{9{,}900}$ **I.** $\frac{1}{10{,}000}$

Short Response

30. **a.** Exercise 29 involves two events. What are they?
   b. Are they independent or dependent? Explain.

31. In a game, Teri and Hector take turns choosing cards at random from a basket. Each card shows a "T" or "H". A player chooses a card, records the letter, and puts the card in a discard pile.
   a. Are the letters drawn independent or dependent events?
   b. Explain your answer.

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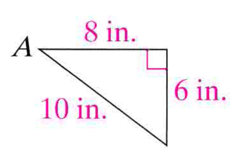
Mixed Review

Lesson 12-4

32. **Travel** From Compt there are four ways to get to Murch. From Murch there are five ways to get to Toll. How many ways are there from Compt to Toll through Murch?

Lesson 11-6

For each triangle, find the sine, cosine, and tangent of angle *A*.

33.


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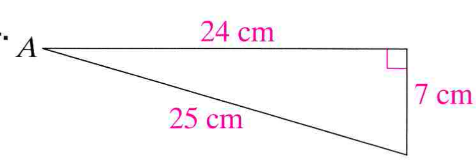


Permutations and Combinations

12-6

OBJECTIVE 1

Permutations

An arrangement in which order is important is a **permutation.** For the letters O, P, S, and T, the permutations *STOP* and *POTS* are different because the order of the letters is different.

You can use the Counting Principle to find the number of possible permutations.

1 EXAMPLE Counting Permutations

Find the number of permutations possible for the letters O, P, S, and T.

1st letter	2nd letter	3rd letter	4th letter		
4 choices	3 choices	2 choices	1 choice		
4 ·	3 ·	2 ·	1	=	24

There are 24 permutations of the letters O, P, S, and T.

Check Understanding Example 1

1. Use the Counting Principle to find the number of permutations possible for the letters W, A, T, E, and R.

A track team has seven members. In how many ways could four team members line up for a relay race?

You can use *permutation notation* to represent this problem.

7 members, Choose 4.

$$_7P_4 = \underset{\text{1st member}}{7} \cdot \underset{\text{2nd member}}{6} \cdot \underset{\text{3rd member}}{5} \cdot \underset{\text{4th member}}{4} = 840$$

Four of seven team members could line up in 840 ways.

Key Concepts Permutation Notation

The expression $_nP_r$ stands for the number of permutations of n objects chosen r at a time.

What You'll Learn

- OBJECTIVE 1 To use permutations
- OBJECTIVE 2 To use combinations

. . . And Why

To solve real-world problems involving sports and geography

Check Skills You'll Need

Use the Counting Principle to find the number of outcomes.

1. Roll 2 number cubes.
2. Choose three different letters.
3. Select a month and a day of the week.
4. Toss a coin 4 times.

For help, go to Lesson 12-4.

New Vocabulary

- permutation
- combination

iTEXT Interactive lesson includes instant self-check, tutorials, and activities.

Graphing Calculator Hint

To evaluate ${}_9P_5$, press 9 **nPr** 5 ENTER. Find **nPr** in the MATH **PRB** menu.

2 EXAMPLE Simplifying Permutation Notation

You have 9 books and want to display 5 on a shelf. How many different 5-book arrangements are possible?

$$\underbrace{{}_{9}}_{\text{9 books}}P_{\underbrace{5}_{\text{Choose 5.}}} = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = 15{,}120$$ **Simplify.**

There are 15,120 arrangements possible.

Check Understanding Example 2

2. Simplify each expression.

a. ${}_5P_2$ **b.** ${}_5P_3$ **c.** ${}_5P_4$ **d.** ${}_5P_5$

OBJECTIVE

2 Combinations

Sometimes the order of items is not important. For instance, a ham and cheese sandwich is the same as a cheese and ham sandwich. An arrangement in which order does not matter is a **combination.**

3 EXAMPLE Real-World Problem Solving

Inland Water

Country	Water Area (mi²)
Australia	26,610
Canada	291,573
Ethiopia	46,680
India	121,391
Tanzania	22,799
United States	79,541

SOURCE: *The Top 10 of Everything*

Geography **In how many ways could you choose two countries from the table when you write reports about inland water?**

Make an organized list of all the combinations.

AC	AE	AI	AT	AU
	CE	CI	CT	CU
		EI	ET	EU
			IT	IU
				TU

Abbreviate by using the first letter of each country's name. First, list all pairs containing Australia. Continue until every pair of countries is listed.

There are fifteen ways to choose two countries from the list of six.

Check Understanding Example 3

3. In how many ways could you choose three different items from a menu containing six items?

Key Concepts Combination Notation

The expression ${}_nC_r$ stands for the number of combinations of n objects chosen r at a time.

In general, there are fewer combinations than permutations. To find the number of combinations, ${}_nC_r$, of r items chosen from n items, find the total number of permutations ${}_nP_r$, and then divide by the number of possible permutations, ${}_rP_r$, for any group of r items.

$${}_nC_r = \frac{{}_nP_r}{{}_rP_r}.$$

4 EXAMPLE Simplifying Combination Notation

You have five choices of sandwich fillings. How many different sandwiches could you make by choosing three of the five fillings?

5 fillings — Choose 3.

$${}_5C_3 = \frac{{}_5P_3}{{}_3P_3}$$

$$= \frac{5 \cdot 4 \cdot 3}{3 \cdot 2 \cdot 1} = 10 \quad \textbf{Simplify.}$$

You could make 10 different sandwiches.

> **Graphing Calculator Hint**
>
> To evaluate ${}_5C_3$, press 5 **nCr** 3 ENTER. Find **nCr** in the MATH **PRB** menu.

Check Understanding Example 4

4. Simplify each expression.

a. ${}_8C_2$ **b.** ${}_8C_3$ **c.** ${}_8C_4$ **d.** ${}_8C_5$

You can tell whether a problem requires permutations or combinations by asking yourself, *Does order matter?* If the answer is *yes,* use permutations. If it is *no,* use combinations.

5 EXAMPLE Identifying Whether Order Is Important

Tell which type of arrangement each problem involves. Explain.

a. How many different groups of three books could you choose from five books?
Combinations; the order of the books selected does not matter.

b. In how many different orders could you play three CDs?
Permutations; the order in which you play the CDs matters.

Check Understanding Example 5

5. Tell which type of arrangment is involved. Explain.

a. A teacher selects a committee of 4 students from 25 students. How many different committees could the teacher select?
b. Class officers are president, vice-president, secretary, and treasurer. From a class of 25 students, how many different groups of officers could students elect?

EXERCISES

For more exercises, see *Extra Practice.*

Practice and Problem Solving

A Practice by Example

Example 1 (page 659)

Use the Counting Principle to find the number of permutations possible for all the letters in each group.

1. S, I, T **2.** P, L, U, S **3.** W, O, R, L, D

Example 2 (page 660)

Simplify each expression.

4. ${}_4P_2$ **5.** ${}_6P_4$ **6.** ${}_9P_4$ **7.** ${}_{10}P_8$

8. How many different arrangements of four books on a shelf could you make from eight books?

Example 3 (page 660)

In how many ways could you choose two different items from each group? Make an organized list of all the combinations.

9. C, A, T **10.** M, A, T, H **11.** V, A, L, U, E

Example 4 (page 661)

Simplify each expression.

12. ${}_4C_2$ **13.** ${}_6C_4$ **14.** ${}_9C_4$ **15.** ${}_{10}C_8$

16. Literature Louisa May Alcott published 13 novels during her lifetime. In how many ways could you select three of these books?

17. You have six choices of sandwich fillings. How many different sandwiches could you make by choosing three of the six fillings?

Example 5 (page 661)

Does each problem involve *permutations* or *combinations?* Explain.

18. In how many different ways could three students form a line?

19. In how many ways could you choose three shirts from seven shirts?

B Apply Your Skills

Find the number of possible 5-letter permutations of the given letters.

20. D, E, C, I, M, A, L **21.** F, A, C, T, O, R **22.** T, R, I, A, N, G, L, E

23. Use the different letters from your last name.
- **a.** Find the number of two-letter permutations.
- **b.** Find the number of two-letter combinations.

Simplify each expression.

24. ${}_6P_3$ **25.** ${}_6C_3$ **26.** ${}_2C_1$ **27.** ${}_2P_1$

28. ${}_{12}P_9$ **29.** ${}_{10}P_5$ **30.** ${}_{10}C_5$ **31.** ${}_{20}C_{19}$

C Challenge

32. Use the letters E, P, S, and T.
- **a.** How many possible arrangements of the letters are there?
- **b.** Add a second letter T to the list. How many distinct arrangements of the five letters are possible?

33. **Writing in Math** To open a combination lock, you turn a dial to match three whole numbers from 0 to 39. You alternate directions of the turns. Explain why 128,000 combinations are possible.

Test Prep

Multiple Choice

34. What is the value of ${}_{10}C_2$?
A. 45 B. 90 C. 180 D. 360

35. A teacher is organizing a class of 24 students into eight groups of three. In how many different ways could she form groups of three?
F. 56 G. 2,024 H. 12,144 I. 735,471

Take It to the NET Online lesson quiz at **www.PHSchool.com** Web Code: ada-1206

Short Response

36. a. How many three-letter permutations are possible for the letters H, E, X, A, G, O, N?
b. How many four-letter permutations are possible?

Mixed Review

Lesson 12-5

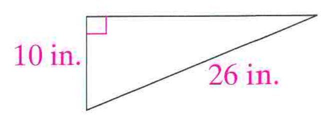
On each of five cards there is one of the letters A, B, C, D, and E. You select two cards. Find *P*(*A*, then *B*) in each situation.

37. with first card replaced

38. with first card not replaced

Lessons 10-2 and 11-2

39. Find the area of the triangle at the right.



Lesson 6-9

40. **Consumer Issues** A coat is on sale for \$80. Its original price was \$120. What is the percent of discount?

Math at Work

Wildlife Statistician

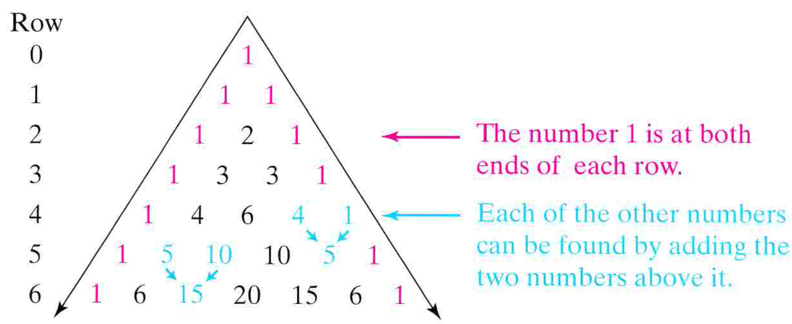
Wildlife statisticians study the growth or decline of plant and animal life in a geographical region. They make observations and collect data for a small portion, or sample, of an animal or plant population. Then they draw conclusions about the entire population. If you enjoy studying wildlife, this may be the job for you.

Take It to the NET For more information about statisticians, go to **www.PHSchool.com.** Web Code: adb-2031

Extension Pascal's Triangle

For Use With Lesson 12-6

The structure of Pascal's triangle is described below.



You can use Pascal's triangle to find combinations.

EXAMPLE

Find $_5C_3$ using Pascal's triangle.

Row 5 of Pascal's triangle gives all the values of $_5C_r$, for $r = 0$ to 5.

1	**5**	**10**	**10**	**5**	**1**	← **row 5**
↕	↕	↕	↕	↕	↕	
$_5C_0$	$_5C_1$	$_5C_2$	$_5C_3$	$_5C_4$	$_5C_5$	← **combinations in the form $_5C_r$, for $r = 0$ to 5**

$$_5C_3 = 10$$

Note that $_nC_0$ equals 1.

EXERCISES

1. Copy and extend Pascal's triangle to Row 10.
2. **a.** Find the sum of the numbers in each row of the triangle.
   b. Write each sum as a power of 2.
   c. Complete: The sum of the numbers in row n is ▒.

Use Pascal's triangle to find each value.

3. $_5C_2$
4. $_3C_2$
5. $_4C_3$
6. $_6C_4$
7. $_7C_2$
8. $_8C_6$
9. $_9C_3$
10. $_9C_5$
11. $_{10}C_0$
12. $_{10}C_{10}$

13. **Reasoning** Explain how to display the values in Row 27 of Pascal's triangle using a calculator.

Experimental Probability

12-7

OBJECTIVE 1

Finding Experimental Probability

Experimental probability is probability based on experimental data.

Key Concepts Experimental Probability

$$P(\text{event}) = \frac{\text{number of times an event occurs}}{\text{number of times experiment is done}}$$

1 EXAMPLE Real-World Problem Solving

Medical Science **A medical study tests a new medicine on 3,500 people. It is effective for 3,010 people. Find the experimental probability that the medicine is effective.**

$$P(\text{event}) = \frac{\text{number of times an event occurs}}{\text{number of times experiment is done}}$$

$$= \frac{3{,}010}{3{,}500} = 0.86, \text{ or } 86\%$$

Check Understanding Example 1

1. Another medicine is effective for 1,183 of 2,275 people. Find the experimental probability that the medicine is effective.

OBJECTIVE 2

Using Simulations

A **simulation** is a model used to find experimental probability.

2 EXAMPLE Using a Simulation

Simulate the correct guessing of true-false answers.

Toss a coin to simulate each guess. Heads represents a correct guess. Here are the results of 50 trials:

HHTTT THHTT THTHH HTTTH TTHTT THTHT
HHTTT HTHHH THTHT THTHT

$$P(\text{heads}) = \frac{\text{number of heads}}{\text{number of tosses}} = \frac{22}{50} = \frac{11}{25}$$

The experimental probability of guessing correctly is $\frac{11}{25}$.

What You'll Learn

- OBJECTIVE 1 To find experimental probability
- OBJECTIVE 2 To use simulations

. . . And Why

To solve real-world problems involving medical science and test-taking

Check Skills You'll Need

Write each decimal or fraction as a percent.

1. 0.8 **2.** 0.53 **3.** 0.625

4. $\frac{3}{5}$ **5.** $\frac{7}{12}$ **6.** $\frac{5}{8}$

For help, go to Lesson 6-5.

New Vocabulary

- experimental probability
- simulation

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✔ Check Understanding Example 2

2. a. In Example 2, compare the experimental probability with the theoretical probability.

b. If you try the experiment 100 times, what is likely to happen to the experimental probability?

More Than One Way

Use theoretical and experimental probabilities to find a probability for correctly guessing all four answers on a four-question true-false quiz.

Nicole's Method

Each guess is independent. Find the probability of one correct guess. Then find the probability of four independent correct guesses.

$P(\text{1 correct guess}) = \frac{1}{2}$

$P(\text{4 correct guesses}) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16}$

The theoretical probability is $\frac{1}{16}$.

Daryl's Method

Simulate the problem by tossing a coin. Let heads stand for a correct guess and tails for an incorrect guess. Use the results of 120 tosses given at the left. Separate the results into 30 groups of 4. Count the groups with 4 heads. There are two.

HTTT	HTTT	HHHT	TTTT	TTHT	THTT
THHH	THHH	HTTT	TTHH	HTTT	HTHH
HTHH	THTT	TTHT	THTH	HHTT	THTH
HHHH	TTHT	HHHH	THTT	TTTH	HHHT
HHTT	HTTT	THTT	HHTT	HHTH	HTHH

Find the experimental probability.

$$P(\text{event}) = \frac{\text{number of times an event occurs}}{\text{number of times experiment is done}}$$

$$= \frac{2}{30} = \frac{1}{15}$$

The experimental probability is $\frac{1}{15}$.

120 Coin Tosses

H T T T H T T T H H H T
T T T T T T H T T H T T
T H H H T H H H H T T T
T T H H H T T T H T H H
H T H H T H T T T T H T
T H T H H H T T T H T H
H H H H T T H T H H H H
T H T T T T T H H H H T
H H T T H T T T T H T T
H H T T H H T H H T H H

For Daryl's simulation, the experimental probability is a little greater than the theoretical probability.

Choose a Method

1. Which method would you use to solve the problem? Explain.

EXERCISES

For more exercises, see *Extra Practice.*

Practice and Problem Solving

A Practice by Example

Example 1 (page 665)

A student randomly selected 68 vehicles in a large parking lot and noted the color of each. Use the results to find the experimental probability that a vehicle chosen at random in the lot is the given color. Write the probability as a percent, to the nearest tenth of a percent.

Color	Number of Vehicles
Black	9
Blue	10
Brown	13
Green	7
Red	12
White	11
Gray	6

1. red

2. white

3. black

4. blue or green

5. not black or gray

6. purple

Example 2 (page 665)

7. In a multiple-choice test, each item has four choices.

a. Tell how to simulate the correct guessing of the correct choice.

b. Carry out your simulation. What do you find for the experimental probability?

c. Compare your result to the theoretical probability.

8. A cereal company randomly puts one of six different stickers in each box of its cereal. Use a simulation to find the experimental probability of getting sticker 3 or sticker 4.

B Apply Your Skills

Students were surveyed about the numbers of pencils in their book bags. The table shows the results. Write each experimental probability as a fraction in simplest form.

Pencils in Students' Book Bags

Number of pencils	Number of students
0	4
1	16
2 or more	12

9. P(one pencil)

10. P(no pencils)

11. P(two or more pencils)

12. P(at least one pencil)

13. a. How would you find an experimental probability for tossing three coins and getting three heads?

b. **Reasoning** How would you compare an experimental probability for getting three heads to the theoretical probability? Would you expect the probabilities to be equal? Explain.

14. **Error Analysis** A student wants to do a simulation to find a probability for correctly guessing a number from 1 to 5 two times in a row. He decides to roll a number cube 100 times, separating the results into 50 groups of two and letting a roll of 1 stand for a correct guess. Explain why the student's simulation will not give good results.

15. Two players played a number-cube game. The table shows the results.
   a. Find P(A wins) and P(B wins).
   b. **Writing in Math** A *fair game* is one in which each player has the same chance of winning. Do you think the game that A and B played is fair? Explain.

Game Results

A Wins	B Wins
𝍸 𝍸 𝍸 𝍸 𝍸 ||||	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ||

Challenge

16. a. **Open-Ended** Write an experimental probability problem that you can solve with a simulation. Your problem should be different from any in this lesson.
   b. Solve the problem.

Test Prep

Gridded Response

17. A baseball manufacturer checked 250 of its baseballs and found that 8 were defective. What is the experimental probability, to the nearest tenth of a percent, that a baseball is NOT defective?

18. Suki tosses two number cubes 100 times. In 19 of the tosses, the sum of the two cubes equals 7. What is Suki's experimental probability of getting a sum of 7 when tossing two number cubes?

19. A medical experiment finds that 232 out of 1,000 patients did NOT respond to the same medication. What is the experimental probability, to the nearest tenth of a percent, that a patient will respond to the medication?

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Web Code: ada-1207

Mixed Review

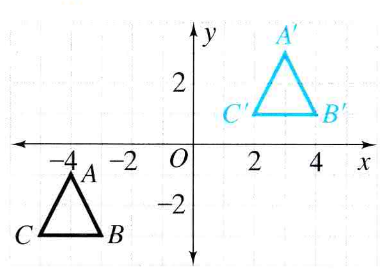
Lesson 12-6 **Evaluate each expression.**

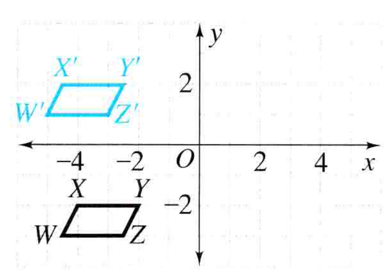
20. ${}_4P_2$ 21. ${}_{10}P_3$ 22. ${}_4C_3$ 23. ${}_6C_3$

Lesson 10-9

24. **Geometry** Find the volume of a spherical globe with a diameter of 0.9 m. Round to the nearest tenth.

Lesson 9-8 **Geometry Write a rule to describe each translation.**

25.


26.


Random Samples and Surveys

12-8

OBJECTIVE 1

Choosing Samples for Surveys

How many books do you read each week? What are your hobbies? Statisticians use questions like these in surveys to get information about specific groups.

A **population** is a group about which you want information. A **sample** is a part of the population you use to make estimates about the population. The larger your sample, the more reliable your estimates will be.

For a **random sample** each member of the population has an equal chance to be selected. A random sample is likely to be representative of the whole population.

1 EXAMPLE Real-World Problem Solving

Recycling **You want to find out whether students will participate if you start a recycling program at your school. Tell whether each survey plan describes a good sample.**

a. **Interview every tenth teenager you see at a mall.**
This sample will probably include students who do not go to your school. It is not a good sample because it is not taken from the population you want to study.

b. **Interview the students in your ecology class.**
The views of students in an ecology class may not represent the views about recycling of students in other classes. This is not a good sample because it is not random.

c. **Interview every tenth student leaving a school assembly.**
This is a good sample. It is selected at random from the population you want to study.

Check Understanding Example 1

1. Explain whether each plan describes a good sample.
   a. You want to know which bicycle is most popular. You plan to survey entrants in a bicycle race.
   b. You want to know how often teens rent videos. You plan to survey teens going into the local video rental store.
   c. You want to know the most popular breakfast cereal. You plan to survey people entering a grocery store.

What You'll Learn

OBJECTIVE 1 To choose a sample for a survey of a population

OBJECTIVE 2 To make estimates about populations

. . . And Why

To solve real-world problems involving recycling and quality control

Check Skills You'll Need

Solve each proportion.

1. $\frac{8}{32} = \frac{n}{450}$
2. $\frac{7}{25} = \frac{n}{24{,}000}$
3. $\frac{2}{250} = \frac{n}{50{,}000}$
4. $\frac{4}{100} = \frac{n}{144{,}000}$

For help, go to Lesson 6-2.

New Vocabulary

- population
- sample
- random sample

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OBJECTIVE

2 Making Estimates About Populations

You can use a sample to make an estimate about a population by writing and solving a proportion.

2 EXAMPLE Real-World Problem Solving

Quality Control **From 20,000 calculators produced, a manufacturer takes a random sample of 500 calculators. The sample has 3 defective calculators. Estimate the total number of defective calculators.**

$$\frac{\text{defective sample calculators}}{\text{sample calculators}} = \frac{\text{defective calculators}}{\text{calculators}}$$ **Write a proportion.**

$$\frac{3}{500} = \frac{n}{20{,}000}$$ **Substitute.**

$$3(20{,}000) = 500n$$ **Write cross products.**

$$\frac{3(20{,}000)}{500} = \frac{500n}{500}$$ **Divide each side by 500.**

$$120 = n$$ **Simplify.**

Estimate: About 120 calculators are defective.

Check Understanding Example 2

2. Use the data in the table below.

Calculator Samples

Sample	Number Sampled	Number Defective
A	500	3
B	200	2
C	50	0

a. Using Sample B, how many of 20,000 calculators would you estimate to be defective?

b. **Reasoning** Would you expect an estimate based on Sample C to be more accurate or less accurate than one based on Sample B? Explain.

c. Explain why you would take a sample rather than counting or surveying an entire population.

In Example 2, you can think of the sample as giving an experimental probability ($\frac{3}{500} = 0.006$). Then you can multiply this probability by the total number of calculators ($0.006 \cdot 20{,}000$) to estimate the total number of defective calculators.

EXERCISES

For more exercises, see *Extra Practice*.

Practice and Problem Solving

A Practice by Example

Sports **You want to find how popular basketball is at your school. State whether each survey plan describes a good sample. Explain.**

Example 1 (page 669)

1. Interview the 10 tallest students in the school.
2. Interview 20 students after picking their ID numbers at random.
3. Interview 30 students watching a basketball game.

Example 2 (page 670)

4. **Quality Control** A worker takes 100 eggs at random from a shipment of 120,000 eggs. The worker finds that four eggs are bad. Estimate the total number of bad eggs.
5. **Estimation** Of 75 pairs of jeans, 7 have flaws. Estimate how many of 24,000 pairs of jeans are flawed.

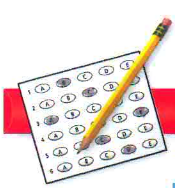
B Apply Your Skills

You want to find which restaurants in your city are most popular. State whether each survey plan describes a good sample. Explain.

6. Choose people to interview at random from the city phone book.
7. Interview every fifth person leaving a restaurant in the city.
8. Interview all the restaurant critics in the state.
9. **Error Analysis** Eight of the 32 students in your math class have a cold. The school population is 450. A student estimates that 112 students in the school have a cold.
   a. Why is your math class not representative of the population?
   b. Describe a survey plan you could use to better estimate the number of students who have a cold.
10. **Reasoning** You survey every fifth student leaving volleyball practice. Of those surveyed, 92% support a proposal to buy new bleachers for the gym. Should you report that there is overwhelming support for the proposal? Explain.

C Challenge

11. **Open-Ended** Describe a survey question, a population, and a sample you could use to make an estimate.
12. **Writing in Math** From 50,000 computer chips produced, you sample 250 chips and find that 0.8% are defective. Explain how you could estimate the total number of defective chips.



Test Prep

Multiple Choice

13. Out of 6,000 radios tested, 12 are defective. What is the estimated total number of defective radios in a group of 250,000?
    A. 60 **B.** 120 **C.** 200 **D.** 500

14. All students at a local elementary school eat lunch at school. You want to find how many students bring their lunches. Which group would be a good sample?
   F. students on one school bus **G.** one first-grade classroom
   H. the cafeteria workers **I.** all third-grade classrooms

Short Response

For Exercises 15 and 16, **(a)** state whether each survey plan describes a good sample. **(b)** Explain your answer.

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15. You want to find how often teens get haircuts. You plan to survey customers in a barbershop.

16. You want to find students' favorite foods. You plan to survey every fourth student leaving the library.

Mixed Review

Lesson 12-7

Use the survey data at the right. Write each experimental probability as a fraction.

17. P(no pets) **18.** P(two or more pets)

19. P(one pet) **20.** P(at least one pet)

Students' Pets

Number of Pets	Number of Students
0	9
1	12
2 or more	5

Lesson 12-6

21. How many combinations of four flowers can you choose from a bouquet of one dozen different flowers?

Lesson 10-3

Geometry **Find the area of each circle. Give the exact area and area rounded to the nearest whole number of units. Use 3.14 for π.**

22. $r = 24$ cm **23.** $d = 45$ in. **24.** $r = 50$ mi

Checkpoint Quiz 2 — Lessons 12-5 through 12-8

Instant self-check quiz online and on CD-ROM

1. A bag contains 10 cards labeled 1–10. You draw one card and then another, without replacing the first card before drawing the second. Find the probability of drawing two even numbers.

2. **a.** A club of 20 students chooses a president and a vice-president. How many different outcomes are possible?
b. A club of 20 students chooses two committee members. How many different committees may be chosen?

3. A hockey player attempts 15 goals and makes 2. Find the experimental probability of making a goal. Predict the number of goals the player will make in the next game if the player attempts 23 goals.

4. Of 450 oranges from a crop of 50,000 oranges, 85 are "premium." Estimate the number of premium oranges in the whole crop.

Technology Using Random Numbers

For Use With Lesson 12-9

Some calculators and computer programs can generate *random numbers.* You can use random numbers for simulations.

To make a list of random integers on a graphing calculator, highlight **randInt** in the MATH **PRB** menu. Press ENTER, and enter "0,9999." Press ENTER repeatedly for a list of four-digit random numbers. Note that the calculator suppresses any zeros at the front of a number. For example, 45 represents the four-digit number 0045.

EXAMPLE

There is a 30% probability of being stopped by a red light at each of four traffic lights. Use a simulation to find an experimental probability of being stopped by at least two red lights.

Use your calculator to generate 20 random 4-digit numbers.

There is a 30% chance of a red light, so let three of the ten digits represent a red light. For this simulation let 1, 2, and 3 represent a red light. Let 4, 5, 6, 7, 8, 9, and 0 represent a yellow or green light.

5186	8918	4275	4285
8124	9619	2517	9964
0912	2759	2329	1666
8938	0357	6755	2227
0201	6325	1905	6885

Any group with two or more of the digits 1, 2, or 3 represents being stopped by at least two red lights. There are seven such groups in this list.

$P(\text{at least two red lights}) = \frac{7}{20}$, or 35%

EXERCISES

1. Use the information in the example. What is the experimental probability of being stopped by exactly three red lights? By four red lights?
2. What would be the result in the example if you had chosen 8, 9, and 0 instead of 1, 2, and 3 to represent red lights?
3. a. Make a new random number list by using randInt(0,999) in place of randInt(0,9999). How many digits are in each random number?
   b. How can you make a 6-digit random number?
4. **Writing in Math** Suppose the probability of being stopped by a red light at each of four lights is 50%. Describe how you would use random numbers to find the probability of getting a red light at two or more lights.
5. About 20% of high school students in the United States say they would like to be president. Use random numbers to find the probability that at least three of the next five high school students you see would like to be president.

12-9 Problem Solving Simulate the Problem

What You'll Learn

OBJECTIVE 1 To solve problems by simulation

. . . And Why

To solve real-world problems involving sports

✓ Check Skills You'll Need

Create a frequency table showing the number of times each letter of the alphabet appears in the sentence below.

A simulation is a model of a real experience.

For help, go to Lesson 12-1.

OBJECTIVE 1 Simulate the Problem

Math Strategies in Action

Do you dream of flying your own airplane? Can you picture yourself in a space shuttle? Flight simulators help pilots train for real flying. Simulators are models of the real experience.

You can use simulations to investigate real-world problems. First develop a model, and then conduct an experiment.

1 EXAMPLE Real-World Problem Solving

Basketball **As time is running out in the basketball game, your team is behind by one point. You are fouled and go to the free-throw line. If you miss the shot, your team loses. If you make it, the score is tied and you get another shot. If you miss the second shot, the game ends in a tie. If you make both shots, your team wins. Your average at free throws is four out of five. Simulate the situation and find an experimental probability for each event.**

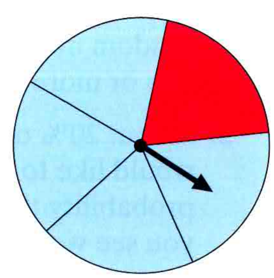
a. You tie the game. b. You win the game. c. You tie or win the game.

Read and Understand

Think about the problem.

1. Based on your average, what is the probability of making one free throw?
2. What methods could you use to simulate the problem?

Plan and Solve



You can use a spinner to simulate the problem. Construct a spinner with five congruent sections. Make four of the sections blue and one of them red. The blue section represents *makes the shot* and the red section represents *misses the shot*. Each spin represents one shot.

Interactive lesson includes instant self-check, tutorials, and activities.

3. How many spins will you make for each experiment?
4. How many experiments will you do?

Use the results given in the table below. "B" stands for blue and "R" stands for red. Note that there is no second shot when the first shot is a miss (R).

Results of 100 Experiments

BR	BB	BB	BB	BB	BR	BB	BB	BB	BB
BR	BB	BR	BB	R	BB	R	BB	BR	BB
BB	BB	BB	R	BB	BB	BB	BB	BB	BB
BB	R	BB	BB	BB	BB	BB	BB	BB	BB
BB	R	BR	BR	BB	BB	BB	R	BB	BR
BB	BR	BB	BB	BB	R	BB	R	BB	BB
BB	R	BR	BB	BB	R	BB	BR	BB	R
BB	BB	BB	BB	BB	BB	BB	BR	BB	BB
BB	R	BR	BB	R	BB	BB	BB	R	BB
BR	BR	BB	R	BB	R	BB	BB	BB	BB

Make a frequency table.

Misses the first shot (R)	Makes the first shot and misses the second shot (BR)	Makes the first shot and makes the second shot (BB)
卌 卌 卌 |	卌 卌 卌	卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 ||||

5. Find each experimental probability.
   a. Your team ties the game.
   b. Your team wins the game.
   c. Your team ties or wins the game.

Look Back and Check

Simulations can give different results. You may find a different probability if you do another simulation. The more experiments you do, the closer the results of different simulations are likely to be.

✓ Check Understanding

6. Continue the simulation with another 100 experiments. Combine the results with the results of the first 100 experiments.
7. Based on the second simulation, what is the probability that your team wins?

EXERCISES

For more exercises, see *Extra Practice.*

Practice and Problem Solving

A Practice by Example

Example 1 (page 674)

Solve by simulating the problem.

1. What is the experimental probability that exactly three children in a family of five children will be boys? Assume that $P(\text{boy}) = P(\text{girl})$.

2. You take a three-question multiple-choice test. Each question has four choices. You don't know any of the answers. What is the experimental probability that you will guess exactly two out of three correctly?

B Apply Your Skills

Solve using any strategy.

3. Thirteen of 25 students are going on a field trip. Six students are traveling in a van. What is the theoretical probability that a student chosen at random from those going on the trip is *not* traveling in the van?

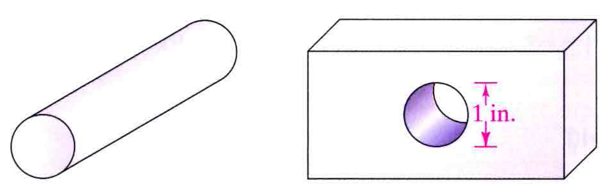
4. A student draws a card at random from the cards below. What is the theoretical probability that the student will draw a card showing A or B?

P R O B A B I L I T Y

5. Prices The original cost of a jacket is $72. During a sale, the store reduces the jacket price by 25%. After the sale, the store raises the reduced jacket price by 25%. What is the price of the jacket after it is increased?

6. Geometry A farmer uses 24 yd of fencing to make a rectangular pen. The pen is 6 yd longer than it is wide. What are the dimensions of the pen?

7. Reasoning The circumference of the peg is 3 in. Will the peg go through the hole? Explain.



8. You toss five coins. What is the theoretical probability that you will get five heads?

9. Many sweepstakes contests have an elimination round. In the elimination round, half of the entrants are chosen at random to go on to the final round. Then one person is chosen as the winner. What is the theoretical probability that a person who enters a contest with 10,000 entrants will be the winner?

Strategies

- Account for All Possibilities
- Draw a Diagram
- Look for a Pattern
- Make a Model
- Make a Table
- Simplify the Problem
- Simulate the Problem
- Solve by Graphing
- Try, Test, Revise
- Use Multiple Strategies
- Work Backward
- Write an Equation
- Write a Proportion

Challenge

10. On a TV game show, you try to win a prize that is hidden behind one of three doors. After you choose a door, but before it is opened, the host opens one of the other doors, behind which there is no prize. You can then switch to the remaining closed door or stay with your original choice.
   a. Find the experimental probability of winning if your strategy is to stay with your original choice. (*Hint:* Simulate by using one marked index card and two unmarked index cards.)
   b. Find the experimental probability of winning if your strategy is to switch to the other door.
   c. **Writing in Math** Should you stay or switch in this game? Explain.

11. Each box of Tastycrunch cereal contains a prize. There are four possible prizes. The prizes are equally likely. You purchase 10 boxes of Tastycrunch. Simulate the problem to find the experimental probability that you will get all four prizes.

Test Prep

Multiple Choice

12. Raul has 4 pairs of shoes to choose from each day. He wants to find the experimental probability of choosing his favorite pair of shoes at random. Which simulation should Raul use?
   A. Tossing a coin
   B. Rolling a number cube
   C. Spinning a spinner with 4 congruent sections
   D. Drawing chips from a bag with 8 different-colored chips

13. Angela conducts a simulation in which she gets a favorable outcome 230 times out of 500 trials. What is the experimental probability of a favorable outcome?
   F. 2.3% **G.** 23% **H.** 46% **I.** 230%

14. Jerome constructs a spinner that has 5 congruent sections. He writes "yes" in one section and "no" in the other four sections. Jerome uses this spinner to simulate making free-throw shots. What experimental probability is he most likely to find?
   A. 1% **B.** 5% **C.** 20% **D.** 80%

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Mixed Review

Lesson 12-8

15. **Quality Control** Six out of every 80 wrenches are found to be defective. For a batch of 3,200 wrenches, estimate the number of wrenches that will *not* have any flaw.

Lesson 11-5

16. **Geometry** Each side of a square kite is to be 20 in. long. To the nearest inch, what length of wood do you need to make the diagonals?

Lesson 11-1

Estimate to the nearest integer.

17. $\sqrt{15}$ 18. $\sqrt{10}$ 19. $\sqrt{50}$ 20. $-\sqrt{82}$

Test-Taking Strategies

Answering True/False Questions

In true/false questions, the entire statement must be true. Otherwise, the statement is false. A statement may be true for many cases and false for just one. This one case is called a *counterexample.* If you think a statement is false, try to find a counterexample.

1 EXAMPLE

True or False? The mean of a set of values is always different from the median of the same set of values.

Choose a simple example that has an odd number of values, and that is "evenly balanced" above and below the middle value.

8, 10, 12, 14, and 16

Find the median: 8, 10, **12**, 14, 16
Find the mean: $(8 + 10 + 12 + 14 + 16) \div 5 = 12$

This is a counterexample to the statement, so the statement is false.

2 EXAMPLE

Suppose two events are independent and the probability of each is less than 1. Is the following statement true or false? The probability that the two events occur is less than the probability of either event.

Since the events are independent, $P(A \text{ and } B) = P(A) \cdot P(B)$. $P(A)$ and $P(B)$ are both less than 1. The product of two (proper) fractions is less than either fraction. Test some cases:

$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ $\quad$ $\frac{1}{8}$ is less than both $\frac{1}{2}$ and $\frac{1}{4}$.

$\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$ $\quad$ $\frac{3}{8}$ is less than both $\frac{3}{4}$ and $\frac{1}{2}$.

It seems likely that $P(A \text{ and } B) < P(A)$ and $P(A \text{ and } B) < P(B)$ are always true.

EXERCISES

Determine whether each statement is *true* or *false*. Explain.

1. The median of a data set is always one of the data values.
2. The number of permutations of two items from a data set is always two times the number of combinations when taking two objects at a time from the same data set.
3. In large data sets (1,000 or more values), the mean is always one of the data values.

Chapter 12

Chapter Review

Vocabulary

box-and-whisker plot (p. 635)	**independent events** (p. 654)	**range** (p. 631)
combination (p. 660)	**line plot** (p. 631)	**sample** (p. 669)
counting principle (p. 650)	**permutation** (p. 659)	**sample space** (p. 650)
dependent events (p. 656)	**population** (p. 669)	**simulation** (p. 665)
experimental probability (p. 665)	**quartiles** (p. 635)	**theoretical probability** (p. 650)
frequency table (p. 630)	**random sample** (p. 669)	

Reading Math
Understanding Vocabulary

Choose the vocabulary term that correctly completes the sentence.

1. An arrangement in which order does not matter is a _?_.
2. The part of a population used to make estimates about the entire population is a _?_.
3. The ratio of the number of favorable outcomes to the number of possible outcomes is the _?_ of an event.
4. An arrangement in which order is important is a _?_.
5. The difference between the greatest and the least values in a data set is the _?_ of the data set.
6. A listing of a data set that shows the number of times each data item occurs is a _?_.
7. Events in which the first event *does* affect the second event are _?_.
8. In a _?_ each member of the population has an equal chance of being selected.

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Skills and Concepts

12-1 Objectives

- To display data in frequency tables (p. 630)
- To display data in line plots (p. 631)

You can show data in a **frequency table,** which lists each data item with the number of times it occurs, or a **line plot,** which displays data with **X** marks on a number line. The **range** is the difference between the greatest and the least values in a set of data.

Display each set of data in a frequency table.

9. 11 10 12 10 12 11 13 12 11 9 12 10
10. 47 48 46 47 45 49 46 48 50 48 46 49

Draw a line plot for each frequency table. Find the range.

11.

Number	1	2	3	4	5	6
Frequency	6	4	5	2	3	1

12.

Number	1	2	3	4	5	6
Frequency	2	8	6	7	3	1

12-2 Objectives

- ▼ To make box-and-whisker plots (p. 635)
- ▼ To analyze data in box-and-whisker plots (p. 636)

A **box-and-whisker plot** displays data items below a number line. **Quartiles** divide the data into four parts. The median is the middle quartile. You can compare two sets of related data by making two box-and-whisker plots on one number line.

Make a box-and-whisker plot for each set of data.

13. 6 9 6 5 8 2 3 9 4 8 5 7 12 9 4

14. 21 35 26 32 24 30 29 38 27 32 51

12-3 Objectives

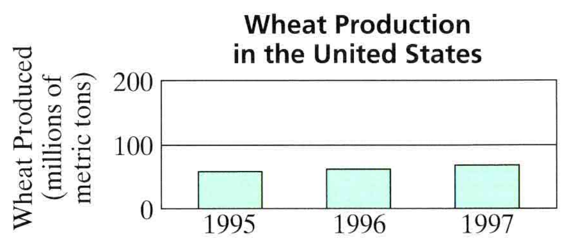
- ▼ To recognize the use of breaks in the scales of graphs (p. 642)
- ▼ To recognize the use of different scales (p. 643)

A graph can give a different impression if a break is used in the scale. A graph can be misleading when a scale is distorted.

For Exercises 15 and 16, use the graph showing wheat production.

15. What does the graph suggest about U.S. wheat production?

16. Explain how you could redraw the graph so that production seems to be increasing dramatically.



12-4 Objectives

- ▼ To use a tree diagram and the Counting Principle to count possible choices (p. 649)
- ▼ To find theoretical probability by counting outcomes (p. 650)

A **sample space** is all the possible outcomes of an event. Use a tree diagram or the **Counting Principle** to count the number of outcomes. You can count outcomes to help find **theoretical probability.**

17. Volunteers have made a large number of sandwiches for a school party. The sandwiches come on white bread, on whole wheat bread, or on a roll. Each contains one of five fillings: turkey, chicken, egg salad, cheese, or peanut butter.
   a. How many different types of sandwich are possible?
   b. There are 20 of each type of sandwich. You receive one sandwich at random. Find the theoretical probability of getting a sandwich on bread with a meat filling.

12-5 Objectives

- ▼ To calculate probabilities of independent events (p. 654)
- ▼ To calculate probabilities of dependent events (p. 656)

Independent events are events for which the occurrence of one event does not affect the probability of the occurrence of the other. If A and B are independent events, the probability that both A and B occur is $P(A \text{ and } B) = P(A) \cdot P(B)$.

Dependent events are events for which the occurrence of one event affects the probability of the occurrence of the other. If A and B are dependent events, the probability that A and then B occur is $P(A, \text{ then } B) = P(A) \cdot P(B \text{ after } A)$.

You select a card at random from those at the right. Find the probability of each event.

E V E N T

18. You select E, replace the card, and then select V.

19. You select T, do not replace the card, and then select N.

12-6 Objectives

- To use permutations (p. 659)
- To use combinations (p. 660)

An arrangement in which order is important is a **permutation.** An arrangement in which order does not matter is a **combination.**

Tell whether each question is a *permutation* or a *combination* problem. Explain. Then find each answer.

20. In how many different ways can five people line up for a photo?

21. How many groups of three pens can you select from a box of twelve pens?

12-7 and 12-9 Objectives

- To find experimental probability (p. 665)
- To use simulations (p. 665)
- To solve problems by simulation (p. 674)

Experimental probability is based on experimental data. You can use a simulation to model real-world problems.

Use the survey data at the right. Write each experimental probability as a fraction in simplest form.

22. P(one notebook)

23. P(at least two notebooks)

Notebooks in Students' Lockers

Number of Notebooks	Frequency
0	1
1	9
2	6
3 or more	4

24. You take a 3-question multiple-choice quiz. Each question has 3 choices. You don't know any of the answers. Use a simulation to find an experimental probability that you will guess exactly 2 out of 3 correctly.

12-8 Objectives

- To choose a sample for a survey of a population (p. 669)
- To make estimates about populations (p. 670)

A **population** is a group about which you want information. A **sample** is a part of the population you use to make estimates for the population. In a **random sample** each member of the population has an equal chance to be selected.

You want to find the favorite brand of in-line skates in your town. Does each survey plan describe a good sample? Explain.

25. You interview students in your homeroom.

26. You interview every tenth student entering the building.

Chapter 12

Chapter Test

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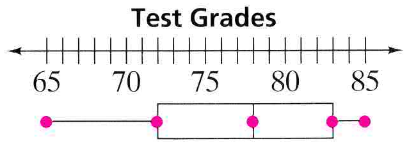
For Exercises 1 and 2, use the box-and-whisker plot.



1. What is the median grade on the test?
2. What is the range in grades?
3. Make a box-and-whisker plot for the data.
75, 70, 80, 85, 85, 55, 60, 60, 65, 85, 75, 95, 50
4. Use the data below.
8, 4, 5, 1, 8, 4, 7, 9, 10, 5, 0, 5, 3, 4, 2
   a. Display the data in a frequency table.
   b. Display the data in a line plot.
   c. Find the range of the data.

For Exercises 5 and 6, use the table. The table shows the money spent on movie tickets.

Year	Dollars (billions)
1994	5.6
1995	6.0
1996	6.3

5. Draw a graph that emphasizes the increase in money spent over time.
6. Draw a graph to suggest that the money spent has not changed much over time.

Use the word TRAIN. Find the probability of each event when a letter is drawn at random.

7. selecting an R, replacing it, and then selecting an N
8. selecting an R, not replacing it, and then selecting an N
9. a. Find the sample space for tossing 3 coins.
   b. Find the theoretical probability of tossing 2 heads and 1 tail.

Simplify each expression.

10. ${}_3P_2$
11. ${}_5C_2$

Find the number of three-letter permutations you can make using each group of letters.

12. F, O, U, R
13. L, U, N, C, H

A student has 4 blue shirts and 2 white shirts. He selects one shirt at random. Without replacing the shirt, he selects a second shirt at random. Find each probability.

14. *P*(blue, then white)
15. *P*(white, then blue)
16. *P*(blue, then blue)
17. *P*(white, then white)

For Exercises 18–21, use the table below. The table shows the colors of a random sample of the bicycles in a rack at school.

Color	Number of Bicycles
Black	9
Blue	10
Red	14

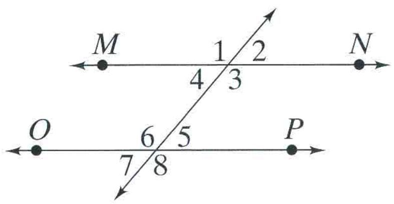
Find each experimental probability for a bicycle chosen at random from the rack. Write each probability as a percent, to the nearest tenth of a percent.

18. *P*(red)
19. *P*(blue)
20. *P*(black)
21. How many bicycles would you expect to be black if there are 50 bicycles in the rack?
22. You roll a pair of number cubes once. What is the probability of rolling doubles?
   a. Find the sample space. Then find the theoretical probability.
   b. Use a simulation to find the experimental probability.
   c. **Writing in Math** For parts (a) and (b), how close should you expect your answers to be? Explain.

Test Prep

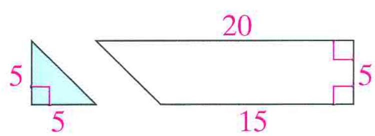
Multiple Choice

1. $\overleftrightarrow{MN} \parallel \overleftrightarrow{OP}$. Which angles are supplementary?



A. $\angle 1$ and $\angle 3$ B. $\angle 4$ and $\angle 6$
C. $\angle 2$ and $\angle 5$ D. $\angle 7$ and $\angle 5$

2. How many shaded triangles will exactly fill the trapezoid?



F. four G. five
H. six I. seven

3. Each of the six faces of a cube is painted either yellow or green. When the cube is tossed, the probability is $\frac{2}{3}$ that the cube will land with a green face up. How many faces are yellow?
A. one B. two
C. three D. four

Gridded Response

4. Simplify ${}_6P_3$.

5. A student has 3 blue T-shirts and 2 white T-shirts. He selects one T-shirt at random. Without replacing the T-shirt, he selects another T-shirt at random. What is the probability that the student will pick a white T-shirt and then a blue T-shirt?

6. A shoe manufacturer checks 150 pairs of walking shoes and finds three pairs to be defective. What is the estimated percent probability that a pair of walking shoes is defective?

Short Response

7. For the data listed below, **(a)** make a box-and-whisker plot. **(b)** Label the median and the lower and upper quartiles.
55, 50, 60, 65, 65, 35, 40, 40, 45, 65, 55, 75, 30, 35, 55, 60, 45, 55

8. Use the data in the table. Make a graph that suggests each situation.
a. sales decreasing sharply
b. sales staying about the same

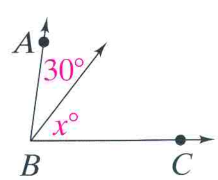
Year	Sales (dollars)
1997	18.2 million
1998	17.9 million
1999	17.7 million
2000	17.5 million

In Exercises 9 and 10, (a) state whether each survey plan describes a good sample. (b) Explain your reasoning.

9. To find how many people who buy stamps are buying them for a collection, you survey people in the post office.

10. To find how popular the U.S. women's soccer team is in your school, you survey all the girls in your English class.

11. $\angle ABC$ is an acute angle.
a. Write an inequality for x.
b. Explain your answer for part (a).



Extended Response

12. You roll two number cubes.
a. What is P(6, then 6)?
b. What is P(odd, then even)?
c. Suppose you roll the two number cubes 50 times in a simulation. You roll double 3s six times. What is the experimental probability of NOT getting double 3s?

World Cup Soccer

Applying Data Analysis Anyone who has ever played soccer knows that controlling a rolling ball with your feet may be fun, but it isn't easy! Perhaps that's why the sport of soccer appeals to so many people of all ages all over the world. Soccer has been played in its current version since about 1863, and international competitions began before 1900. Today, Men's and Women's World Cup competitions are held every four years, one year apart.

Team Grin

Players from the United States women's soccer team pose with their gold medals after defeating China to win the 1999 Women's World Cup.

Final Gold

Mia Hamm (left) of the United States and Daniela Alves of Brazil get ready to kick the ball during the 2000 Women's Gold Cup Finals.

Take It to the NET For more information about soccer, go to **www.PHSchool.com**.
Web Code: ade-1253

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FIFA World Cup™ Trophies

The Men's and Women's World Cup trophies are both made of gold. The United States won the FIFA Women's World Cup in 1999. Brazil won the Men's FIFA World Cup™ in 2002.

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Following the Ball

Senegal midfielder Papa Bouba Diop (left) and Swedish forward Henrik Larsson go for the ball.

Activity

Use the table to answer the questions.

1. Make a line plot of the goals scored by the players.
2. What is the range of the data in the table?
3. Find the mean, median, and mode of the data. Which measure best represents the data? Explain.
4. **a.** Based on the data at the right, make a table showing the number of goals scored by each team.
   b. Find the mean and the mode of the data in your table.
   c. Why is the mean of this data different from the mean found in Exercise 3?
5. **Number Sense** Suppose you did not know which team won the 2002 FIFA World Cup™. From the data in the table, can you determine the top two teams? Explain.

2002 FIFA Men's World Cup™ Top Goal Scorers

Flag	Player	Team	Goals Scored
	Ronaldo	Brazil	8
	Rivaldo	Brazil	5
	Miroslav Klose	Germany	5
	Jon Dahl Tomasson	Denmark	4
	Christian Vieri	Italy	4
	Marc Wilmots	Belgium	3
	Pauleta	Portugal	3
	Papa Bouba Diop	Senegal	3
	Ilhan Mansiz	Turkey	3
	Robbie Keane	Ireland	3
	Michael Ballack	Germany	3
	Fernando Morientes	Spain	3
	Raul	Spain	3
	Henrik Larsson	Sweden	3

Where You've Been

- In Chapter 1, you wrote rules for patterns using inductive reasoning.
- In Chapter 2, you learned to simplify variable expressions by combining like terms.
- In Chapter 4, you learned how to multiply powers with the same base by adding the exponents.
- In Chapter 8, you graphed linear functions by making a table of values to show ordered-pair solutions.

Instant self-check online and on CD-ROM

Diagnosing Readiness

(For help, go to the lesson in green.)

Evaluating Expressions (Lesson 1-3)

Evaluate each expression.

1. $8b$, for $b = 5$ **2.** $(-h)^5$, for $h = 2$ **3.** $19 - (n - 6)$, for $n = 8$

4. $4a + 4$, for $a = 6$ **5.** n^2, for $n = 0.8$ **6.** $55 - 3mn$, for $m = 2, n = 5$

7. $\frac{120}{s + r}$, for $s = 25$ and $r = 35$ **8.** $\frac{j - k}{9}$, for $j = 75$ and $k = 12$

Using the Distributive Property (Lesson 2-2)

Simplify each expression.

9. $(d - 4)3$ **10.** $5(3x + 1)$ **11.** $3(u - 8)$ **12.** $-4(-2y - 7)$

13. $4(-3d + 1)$ **14.** $10(5 - 3s)$ **15.** $-3(7 - 2w)$ **16.** $(9 - 2b)3$

Simplifying Variable Expressions (Lesson 2-3)

Simplify each expression.

17. $5a - 4 + 6a$ **18.** $x - 4x + 3x + 5$ **19.** $g + 4 - 3g + g$

20. $5t + 5s + 5t$ **21.** $9b - 3d + 7d - 2b$ **22.** $-4(9c) + 2(-4c) - c$

Equations With Two Variables (Lesson 8-2)

Find the y values of each equation for $x = -2, 0$, and 2.

23. $y = 3x - 4$ **24.** $y = -3x$ **25.** $y = 4x - 2$ **26.** $y = \frac{3}{5}x - 5$

27. $y = 6 - 2x$ **28.** $y = -\frac{1}{4}x - 8$ **29.** $y = \frac{1}{2}x$ **30.** $y = -3x - 1$